

1. IDEALS AND VARIETIES

- 1. Affine algebraic varieties and ideals.** The notion of algebraic variety. The ideal in the ring of polynomials and its connection to a variety.
- 2. Hilbert's basis theorem.** Noetherian rings. Relations between varieties and their reflection in the relations between ideals.
- 3. Nullstellensatz.** The radical of an ideal. Equivalent formulations of Nullstellensatz.

2. FOUNDATIONS OF ALGEBRAIC GEOMETRY

- 4. Zariski topology.** Closed sets. Irreducible varieties. Decomposition of a variety into irreducibles.
- 5. Functions on affine algebraic varieties, maps between varieties.** Coordinate ring of a variety. Morphisms and rational maps. Rational varieties.

3. COMPUTATIONAL METHODS OF ALGEBRAIC GEOMETRY

- 6. Gröbner basis of an ideal.** Monomial ordering. Definition of a Gröbner basis. Buchberger's algorithm for computing a Gröbner basis.
- 7. Resultants.** Definition and properties of the resultant of two polynomials.
- 8. Elimination of variables.** Eliminating variables from a system of polynomial equations using Gröbner bases and using resultants.
- 9. Sturm's theorem.** Bounding real roots of a polynomial. Number of real roots of a polynomial on a given interval.