Form-factors

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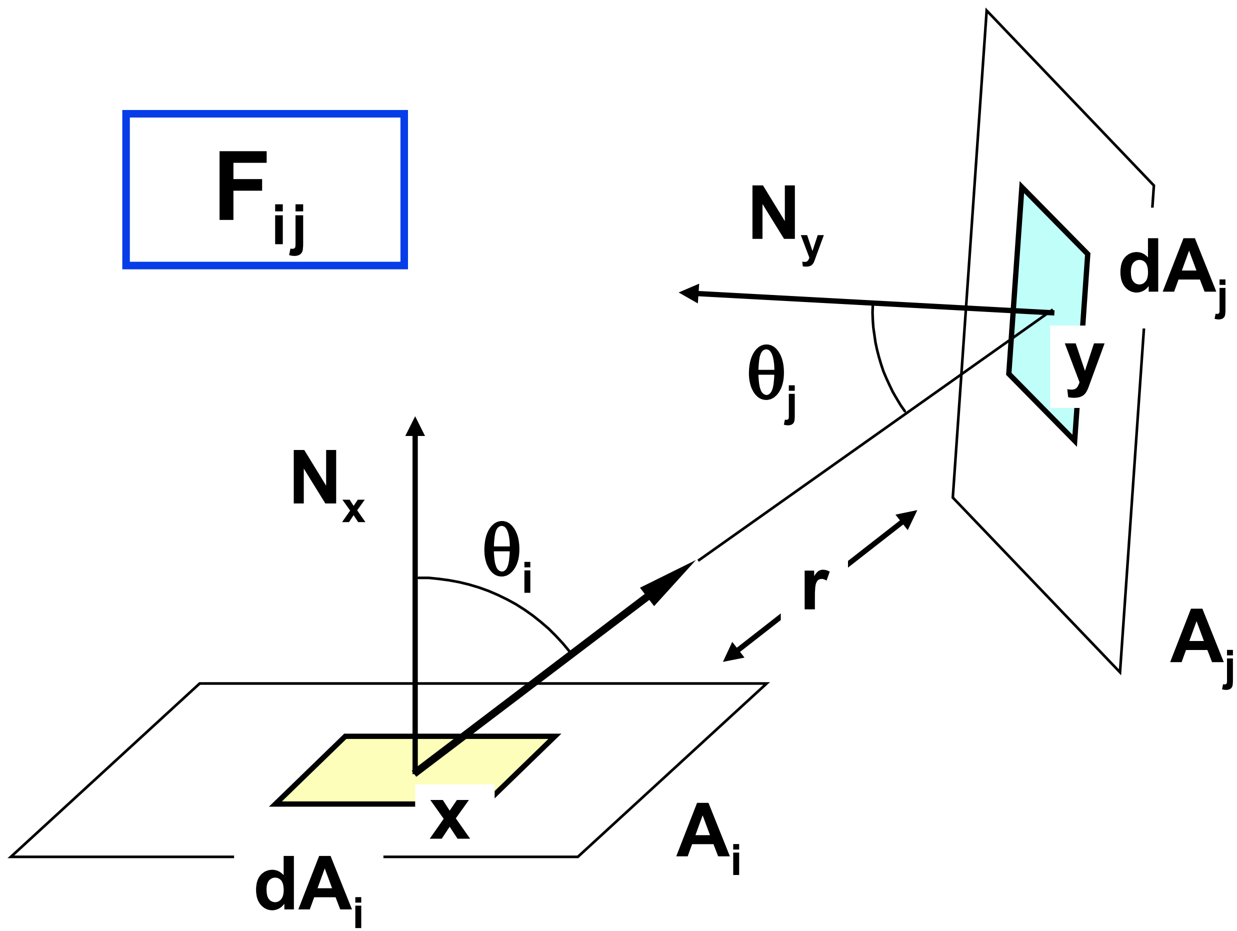
Form-factor F_{ij}

- It indicates the proportion of energy emitted from the surface **i** which will hit the the surface **j**
  - key value when creating a system of linear equations (searching for individual area radiosities)
  - first calculation (physics): Lambert 1760
- it depends only on the **geometry of the scene**
  - distance, inclination and slope of the areas
- F_{ij} is a dimensionless number from the interval $\langle 0,1 \rangle$
  - for a convex polygon **i** is $F_{ii} = 0$

Form-factor



Form-factor

Radiosity equation (with constant elements):

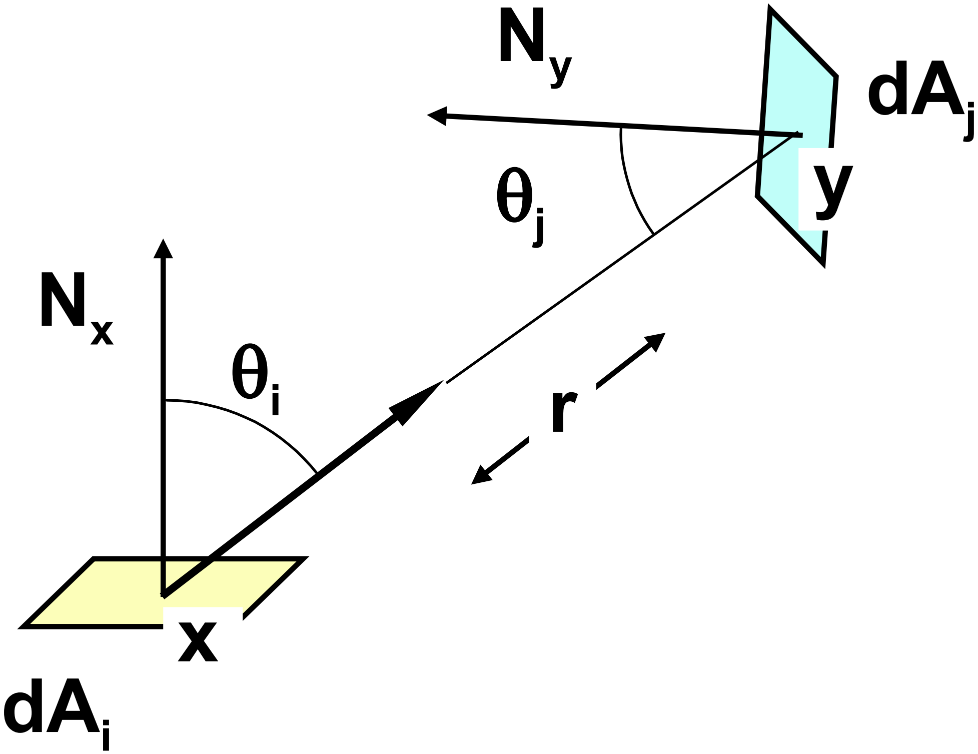
$$B_i = B_{e,i} + \rho_i \cdot \sum_{j=1}^{N} B_j \cdot \boxed{\frac{1}{A_i} \int_{A_i} \int_{A_j} G(y,x)\ dA_j\ dA_i}$$

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} G(y,x)\ dA_j\ dA_i =$$

$$\boxed{= \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cdot \cos\theta_j}{\pi \left\| x - y \right\|^2} \cdot V(x,y)\ dA_j\ dA_i}$$

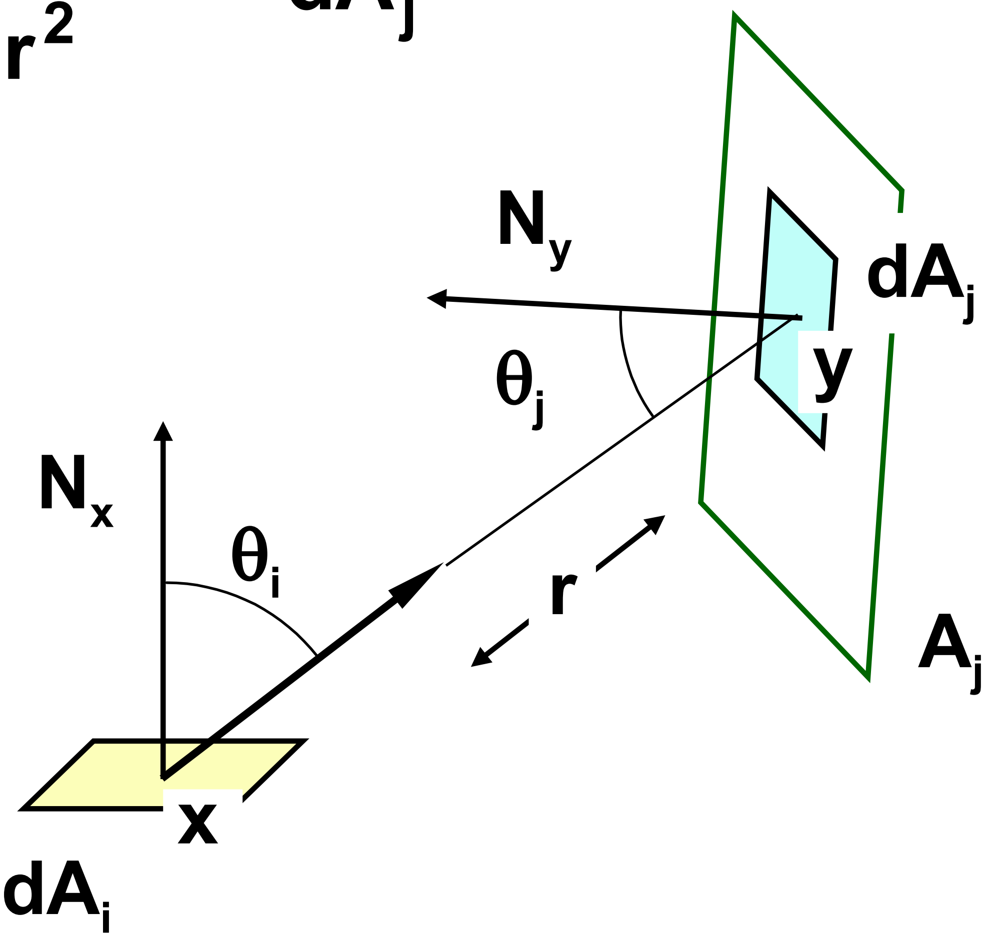
Differential form-factor

$$dF_{dA_i \to dA_j} = \frac{\cos\theta_i \cdot \cos\theta_j}{\pi r^2}$$



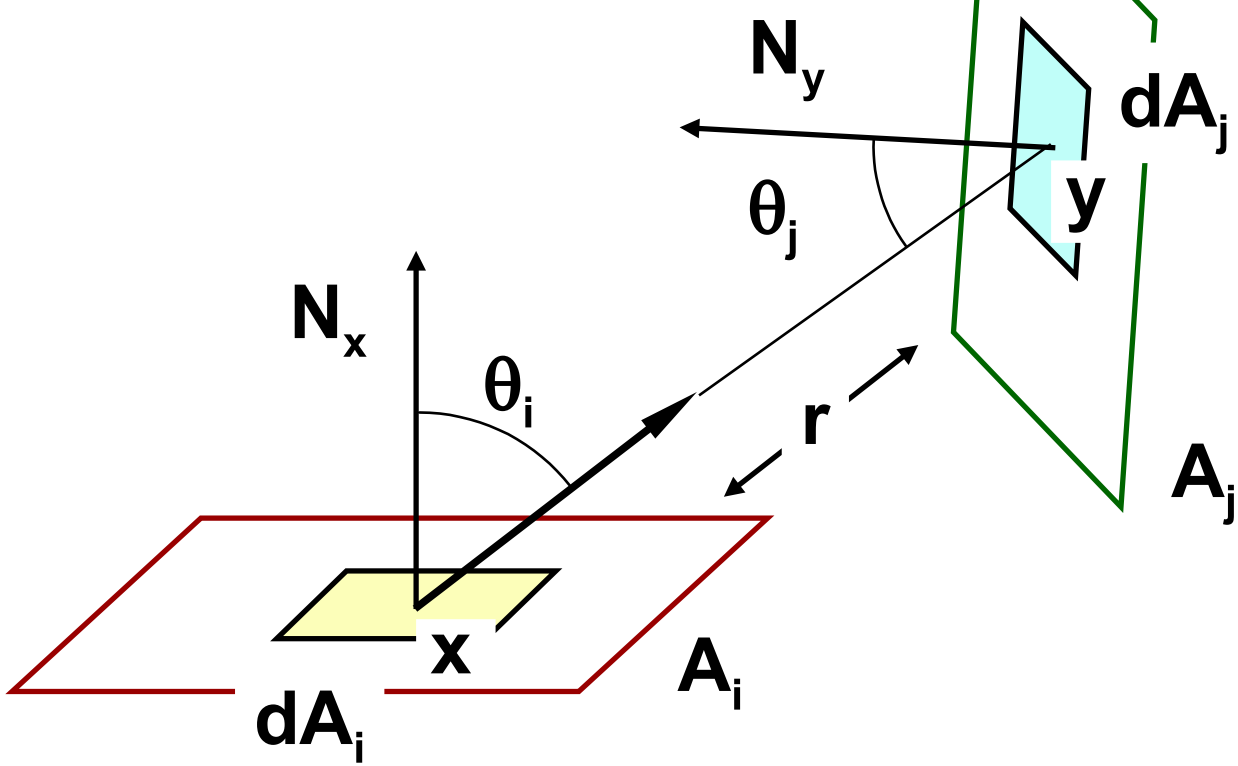
Differential form-factor

$$F_{dA_i \to A_j} = \int_{A_j} \frac{\cos\theta_i \cdot \cos\theta_j}{\pi r^2} \, dA_j$$



Average form-factor

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cdot \cos\theta_j}{\pi r^2} \, dA_j \, dA_i$$



Calculation of form-factors

❶ **analytical methods**

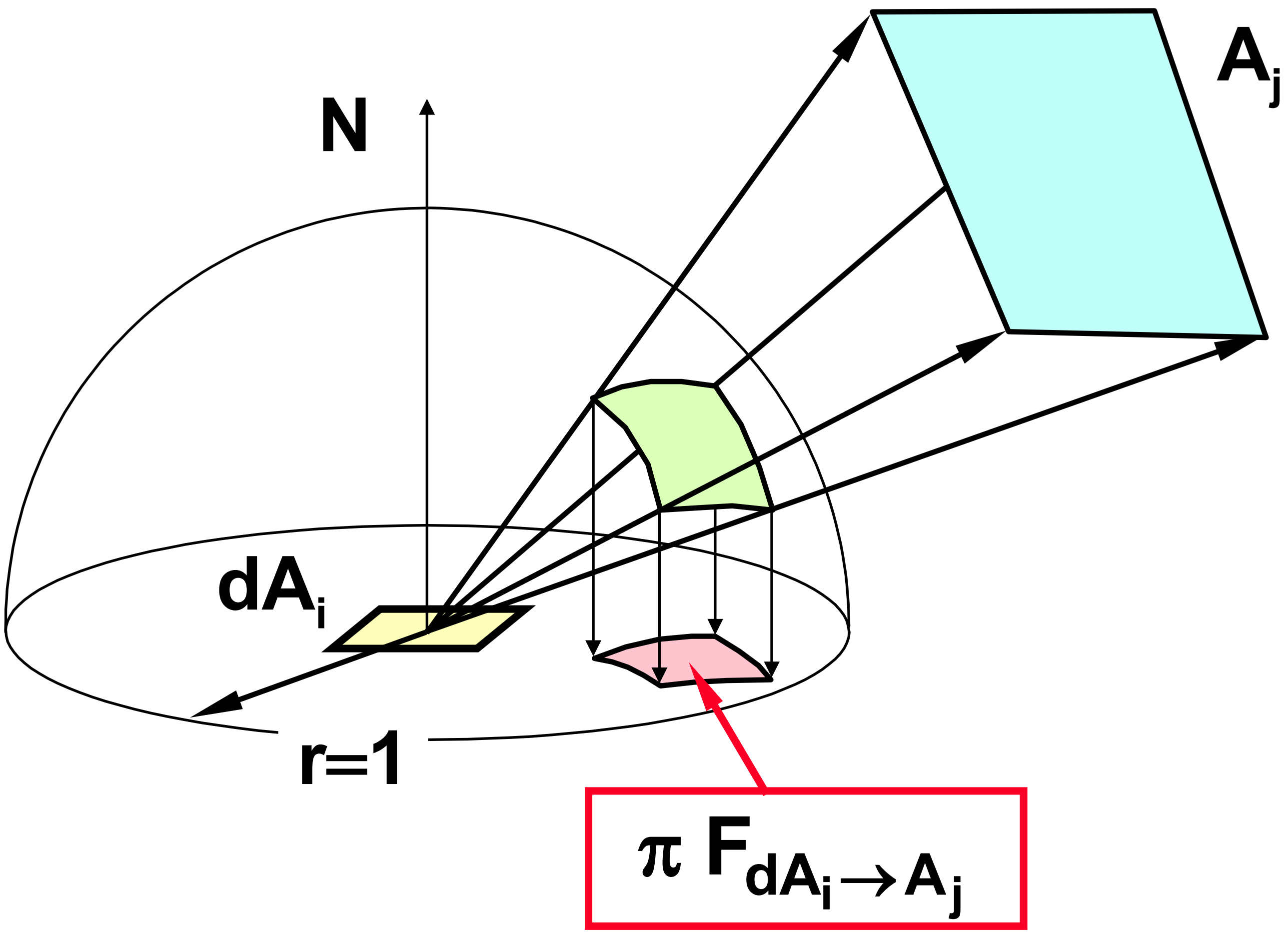
❷ **numerical methods**

– hemicube (Nusselt analogue), projection into one plane, curve integral (according to Stokes' theorem)

❸ numerical **stochastic methods** (Monte-Carlo)

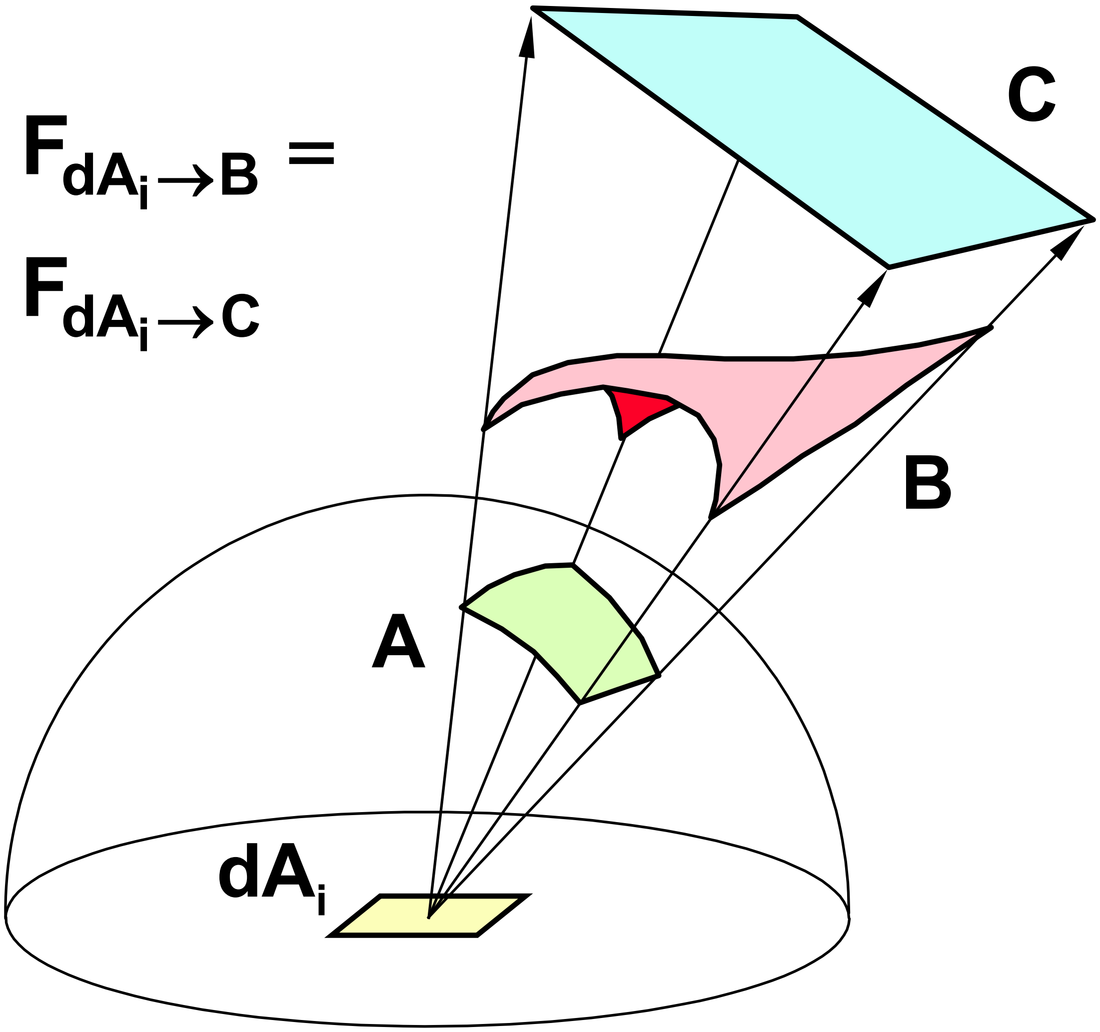
– sampling of a spatial angle or an area that receives energy

Nusselt analogue



Nusselt analogue

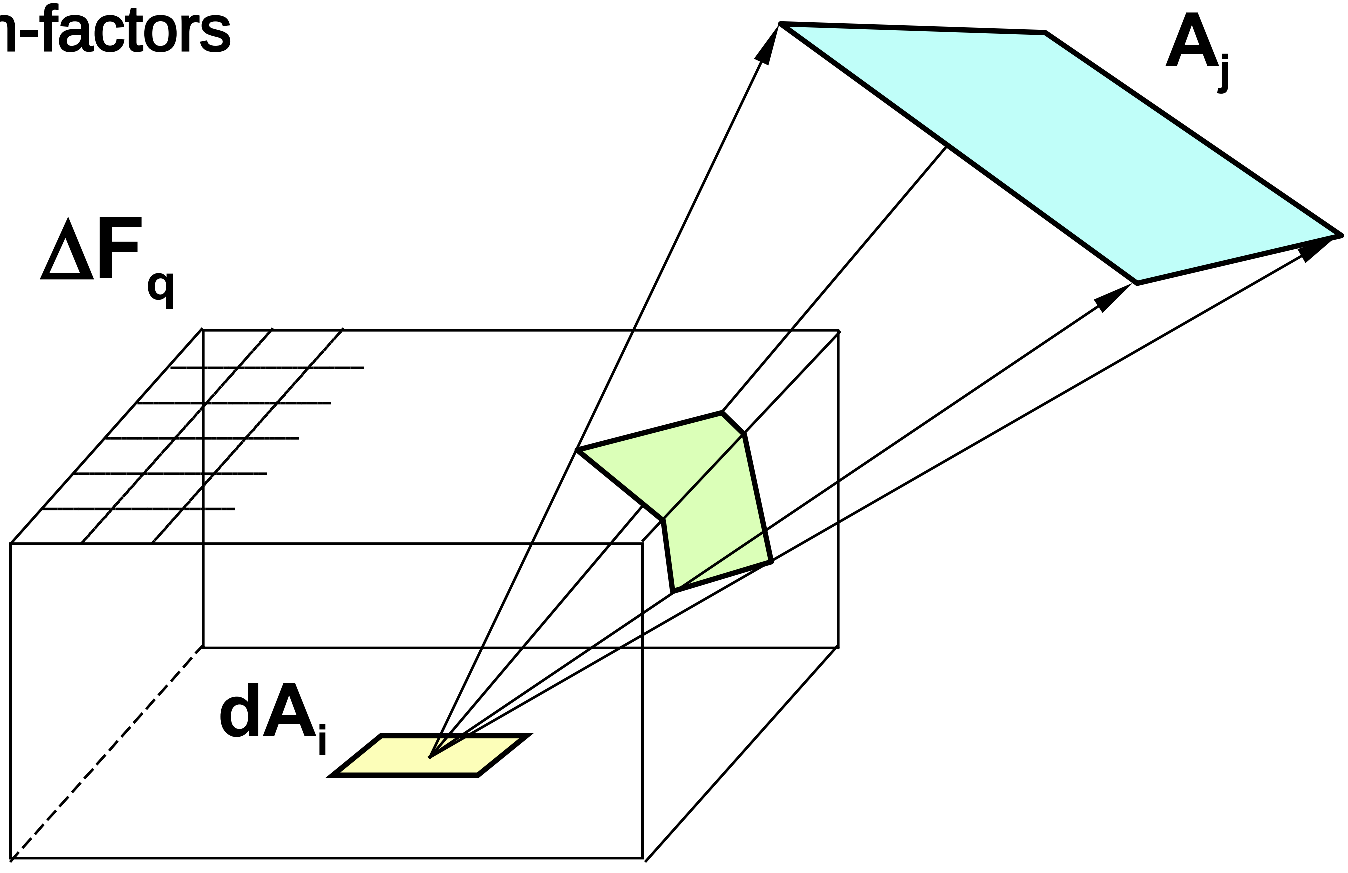
$$F_{dA_i \to A} = F_{dA_i \to B} =$$
$$= F_{dA_i \to C}$$



Hemicube

Regular cell network:

delta form-factors



Hemicube

- calculation of all $\mathbf{F}_{\mathbf{dA_i} \rightarrow \mathbf{A_j}}$ for given **i**
  - we project all other faces $\mathbf{A_j}$ on the hemicube built around $\mathbf{dA_i}$
- we calculate the visibility of the individual faces on the surface of the hemicube (Z-buffer method)
- the surface of the hemicube is divided into a **regular network of cells** $\mathbf{C_q}$
  - for each cell we have calculated delta form-factor $\mathbf{\Delta F_q}$ in advance

Hemicube

- configuration factor $\mathbf{A_j}$ is estimated by cells that were covered by its projection:

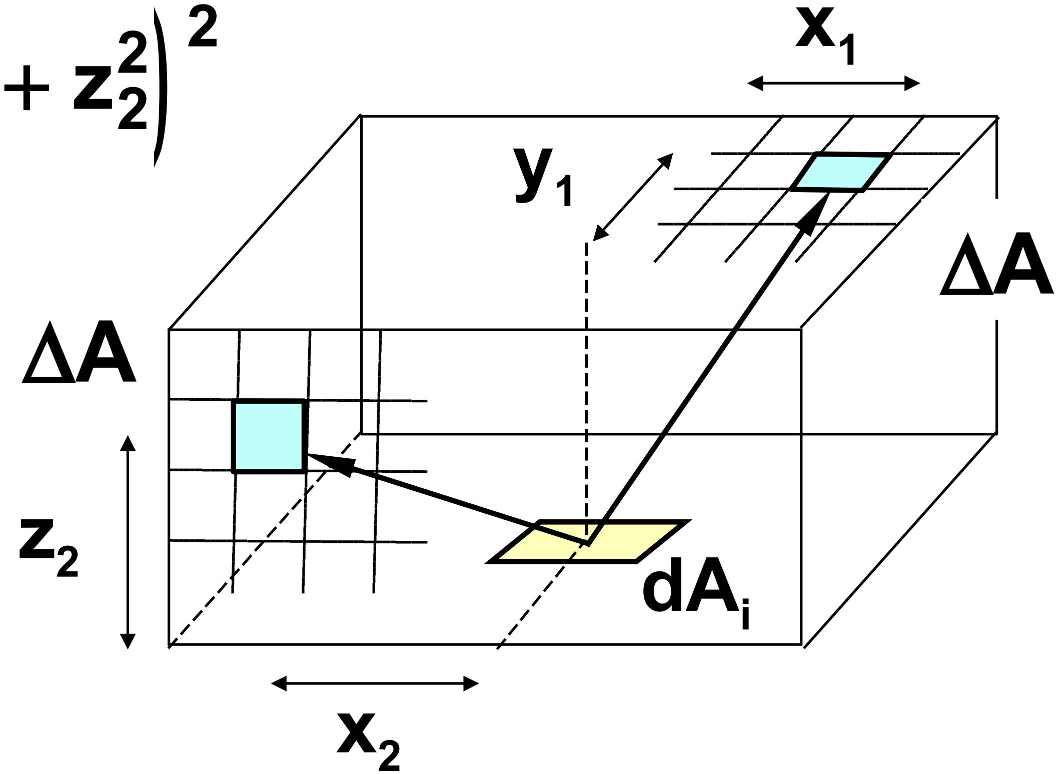
$$F_{dA_i \to A_j} \cong \sum_{q \in J} \Delta F_q$$

- dividing the cube affects the accuracy of the estimation of the configuration factors
  – in practice from **64×64** to **2k×2k** cells were used

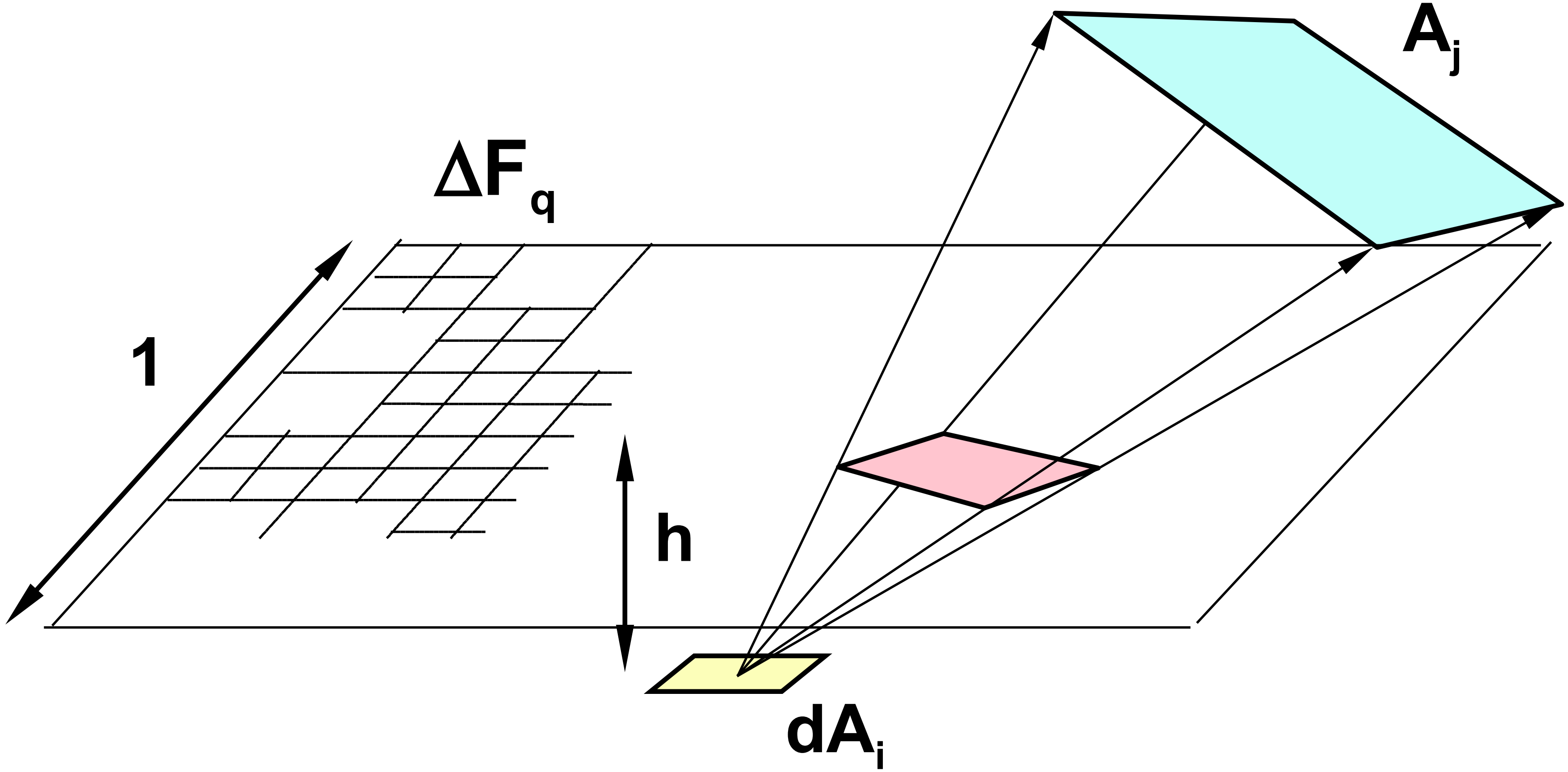
Delta form-factors

$$\Delta F_1 = \frac{\Delta A}{\pi \cdot \left(x_1^2 + y_1^2 + 1 \right)^2}$$

$$\Delta F_2 = \frac{z_2 \cdot \Delta A}{\pi \cdot \left(x_2^2 + 1 + z_2^2 \right)^2}$$



Sillion hemiplane method



Hemiplane method

- **faster implementation** (projection, trimming)
  - part of the spatial angle is neglected
  - the height of the projection plane should be maximum **0.1**
- visibility is calculated by **divide and conquer** method
  - Warnock algoritm analogue
  - adaptive projection plane division $\Rightarrow$ better efficiency
- **delta form-factor** are precalculated for different levels of division

Monte-Carlo methods

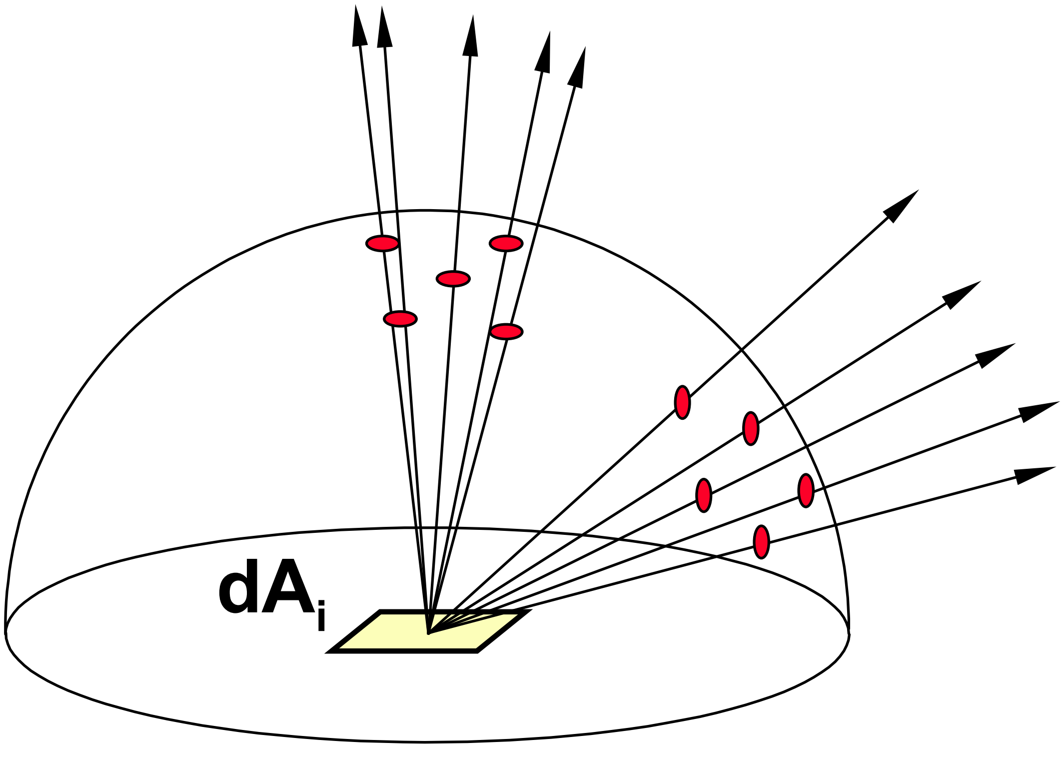
- using the **ray tracing** method
  - it is possible to use more complex scene geometry
  - classic acceleration calculation methods

- sampling of the **object surfaces**
  - calculation of individual form-factor
  - easy calculation for area → area form-factor (independent sampling)

- sampling of the **solid angle**
  - all form-factors from one point at a time

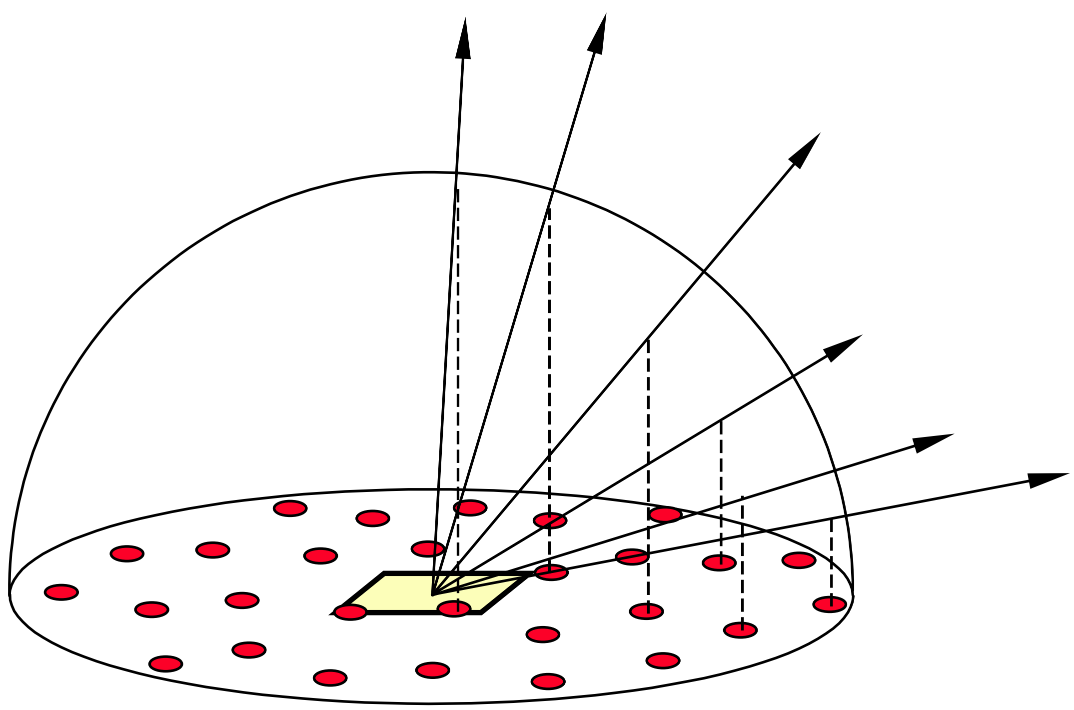
Sampling on the hemisphere

Uniform sampling of the solid angle with **cos** θ_k pdf



Sampling on the source

Non-uniform sampling of the solid angle
all rays have the same importation !



Literature

- **A. Glassner**: ***Principles of Digital Image Synthesis***, Morgan Kaufmann, 1995, 916-937
- **M. Cohen, J. Wallace**: ***Radiosity and Realistic Image Synthesis***, Academ. Press, 1993, 65-107
- **J. Foley, A. van Dam, S. Feiner, J. Hughes**: ***Computer Graphics, Principles and Practice***, 795-799