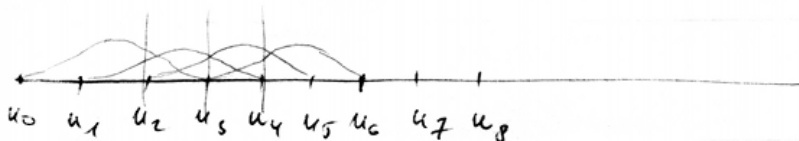


Príklad: Nech $V_0(1,2)$ $V_1(3,5)$ $V_2(6,2)$ $V_3(9,4)$ sú
 vrcholy nadiachto polygónu. Napíšte parametrické
 rovnice B-splínu kvinty stupňa $f=2$

1. Množina uzlov: $U = \{u_0, u_1, u_2, u_3, u_4, u_5, u_6\}$ $m = 3 + 2 + 1 = 6$

$$S(u) = \sum_{i=0}^3 \bar{V}_i N_{i2}(u) = \bar{V}_0 N_{02}(u) + \bar{V}_1 N_{12}(u) + \bar{V}_2 N_{22}(u) + \bar{V}_3 N_{32}(u)$$

$$u \in \langle u_i, u_{i+1} \rangle$$



$$f=0 \quad N_{0,0}(u) = \begin{cases} 1 & u \in \langle u_0, u_1 \rangle \\ 0 & \text{otherwise} \end{cases}$$

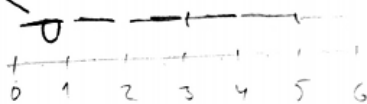
$$N_{3,0}(u) = \begin{cases} 1 & u \in \langle u_3, u_4 \rangle \\ 0 & \text{otherwise} \end{cases}$$

$$N_{1,0}(u) = \begin{cases} 1 & u \in \langle u_1, u_2 \rangle \\ 0 & \text{otherwise} \end{cases}$$

$$N_{4,0}(u) = \begin{cases} 1 & u \in \langle u_4, u_5 \rangle \\ 0 & \text{otherwise} \end{cases}$$

$$N_{2,0}(u) = \begin{cases} 1 & u \in \langle u_2, u_3 \rangle \\ 0 & \text{otherwise} \end{cases}$$

$$N_{5,0}(u) = \begin{cases} 1 & u \in \langle u_5, u_6 \rangle \\ 0 & \text{otherwise} \end{cases}$$



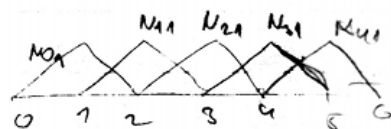
$$f=1 \quad N_{0,1}(u) = \frac{u-u_0}{u_1-u_0} N_{0,0}(u) + \frac{u_2-u}{u_2-u_1} N_{1,0}(u)$$

$$N_{1,1}(u) = \frac{u-u_1}{u_2-u_1} N_{1,0}(u) + \frac{u_3-u}{u_3-u_2} N_{2,0}(u)$$

$$N_{2,1}(u) = \frac{u-u_2}{u_3-u_2} N_{2,0}(u) + \frac{u_4-u}{u_4-u_3} N_{3,0}(u)$$

$$N_{3,1}(u) = \frac{u-u_3}{u_4-u_3} N_{3,0}(u) + \frac{u_5-u}{u_5-u_4} N_{4,0}(u)$$

$$N_{4,1}(u) = \frac{u-u_4}{u_5-u_4} N_{4,0}(u) + \frac{u_6-u}{u_6-u_5} N_{5,0}(u)$$



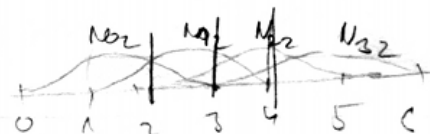
$$f=2 \quad N_{0,2}(u) = \frac{u-u_0}{u_2-u_0} N_{0,1}(u) + \frac{u_3-u}{u_3-u_1} N_{1,1}(u)$$

$$N_{1,2}(u) = \frac{u-u_1}{u_3-u_1} N_{1,1}(u) + \frac{u_4-u}{u_4-u_2} N_{2,1}(u)$$

$$N_{2,2}(u) = \frac{u-u_2}{u_4-u_2} N_{2,1}(u) + \frac{u_5-u}{u_5-u_3} N_{3,1}(u)$$

$$N_{3,2}(u) = \frac{u-u_3}{u_5-u_3} N_{3,1}(u) + \frac{u_6-u}{u_6-u_4} N_{4,1}(u)$$

maximálne \rightarrow



$f = 2$

$$N_{02}(u) = \frac{u-u_0}{u_2-u_0} N_{01}(u) + \frac{u_3-u}{u_3-u_1} N_{11}(u) = \frac{u-u_0}{u_2-u_0} \left[\frac{u-u_0}{u_1-u_0} N_{00}(u) + \frac{u_2-u}{u_2-u_1} N_{10}(u) \right] + \frac{u_3-u}{u_3-u_1} \left[\frac{u-u_1}{u_2-u_1} N_{10}(u) + \frac{u_3-u}{u_3-u_2} N_{20}(u) \right]$$

$$N_{12}(u) = \frac{u-u_1}{u_3-u_1} N_{11}(u) + \frac{u_4-u}{u_4-u_2} N_{21}(u) = \frac{u-u_1}{u_3-u_1} \left[\frac{u-u_1}{u_2-u_1} N_{10}(u) + \frac{u_3-u}{u_3-u_2} N_{20}(u) \right] + \frac{u_4-u}{u_4-u_2} \left[\frac{u-u_2}{u_3-u_2} N_{20}(u) + \frac{u_4-u}{u_4-u_3} N_{30}(u) \right]$$

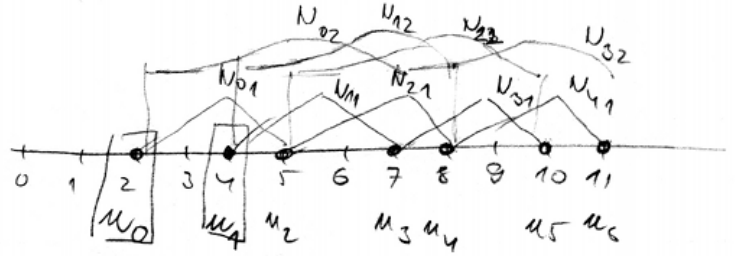
$$N_{22}(u) = \frac{u-u_2}{u_4-u_2} N_{21}(u) + \frac{u_5-u}{u_5-u_3} N_{31}(u) = \frac{u-u_2}{u_4-u_2} \left[\frac{u-u_2}{u_3-u_2} N_{20}(u) + \frac{u_4-u}{u_4-u_3} N_{30}(u) \right] + \frac{u_5-u}{u_5-u_3} \left[\frac{u-u_3}{u_4-u_3} N_{30}(u) + \frac{u_5-u}{u_5-u_4} N_{40}(u) \right]$$

$$N_{32}(u) = \frac{u-u_3}{u_5-u_3} N_{31}(u) + \frac{u_6-u}{u_6-u_4} N_{41}(u) = \frac{u-u_3}{u_5-u_3} \left[\frac{u-u_3}{u_4-u_3} N_{30}(u) + \frac{u_5-u}{u_5-u_4} N_{40}(u) \right] + \frac{u_6-u}{u_6-u_4} \left[\frac{u-u_4}{u_5-u_4} N_{40}(u) + \frac{u_6-u}{u_6-u_5} N_{50}(u) \right]$$

Vypočítajte kmerňaracie funkcie pre veľký vektor:

$$u_0 = 2 \quad u_1 = 4 \quad u_2 = 5 \quad u_3 = 7 \quad u_4 = 8 \quad u_5 = 10 \quad u_6 = 11$$

- nerovnomerný



$$\begin{aligned} \underline{N_{02}(u)} &= \frac{u-2}{5-2} \cdot N_{01}(u) + \frac{7-u}{7-4} N_{11}(u) = \frac{u-2}{3} \left[\frac{u-2}{4-2} N_{00}(u) + \frac{5-u}{5-4} N_{10}(u) \right] + \\ &+ \frac{7-u}{3} \left[\frac{u-4}{5-4} N_{10}(u) + \frac{7-u}{7-5} N_{20}(u) \right] = \\ &= \frac{1}{6} (u-2)^2 N_{00}(u) + \frac{1}{3} (-2u^2 + 18u - 38) N_{10}(u) + \frac{1}{6} (7-u)^2 N_{20}(u) \end{aligned}$$

$$\begin{aligned} \underline{N_{12}(u)} &= \frac{u-4}{3} N_{11}(u) + \frac{8-u}{3} N_{21}(u) = \frac{u-4}{3} \left[(u-4) N_{10}(u) + \frac{7-u}{2} N_{20}(u) \right] + \\ &+ \frac{8-u}{3} \left[\frac{u-5}{2} N_{20}(u) + (8-u) N_{30}(u) \right] = \\ &= \frac{1}{3} (u-4)^2 N_{10}(u) + \frac{1}{3} (-u^2 + 12u - 34) N_{20}(u) + \frac{1}{3} (8-u)^2 N_{30}(u) \end{aligned}$$

$$\begin{aligned} \underline{N_{22}(u)} &= \frac{u-5}{3} N_{21}(u) + \frac{10-u}{3} N_{31}(u) = \frac{u-5}{3} \left[\frac{u-5}{2} N_{20}(u) + (8-u) N_{30}(u) \right] + \\ &+ \frac{10-u}{3} \left[\frac{u-7}{1} \cdot N_{30}(u) + \frac{10-u}{2} N_{40}(u) \right] = \\ &= \frac{1}{6} (u-5)^2 N_{20}(u) + \frac{1}{3} (-2u^2 + 30u - 110) N_{30}(u) + \\ &+ \frac{1}{6} (10-u)^2 N_{40}(u) \end{aligned}$$

$$\begin{aligned} \underline{N_{32}(u)} &= \frac{u-7}{3} N_{31}(u) + \frac{11-u}{3} N_{41}(u) = \frac{u-7}{3} \left[(u-7) N_{30}(u) + \frac{10-u}{2} N_{40}(u) \right] + \\ &+ \frac{11-u}{3} \left[\frac{u-8}{2} N_{40}(u) + (11-u) N_{50}(u) \right] = \\ &= \frac{1}{3} (u-7)^2 N_{30}(u) + \frac{1}{3} (-u^2 + 18u - 79) N_{40}(u) + \frac{1}{3} (11-u)^2 N_{50}(u) \end{aligned}$$

$$N_{02}(u) = \begin{cases} \frac{1}{6}(u-2)^2 & u \in \langle 2, 4 \rangle \\ \frac{1}{3}(-2u^2 + 18u - 38) & u \in \langle 4, 5 \rangle \\ \frac{1}{6}(7-u)^2 & u \in \langle 5, 7 \rangle \\ 0 & u \notin \langle 2, 7 \rangle \end{cases}$$

$$N_{12}(u) = \begin{cases} \frac{1}{3}(u-4)^2 & u \in \langle 4, 5 \rangle \\ \frac{1}{3}(-u^2 + 12u - 34) & u \in \langle 5, 7 \rangle \\ \frac{1}{3}(8-u)^2 & u \in \langle 7, 8 \rangle \\ 0 & u \notin \langle 4, 8 \rangle \end{cases}$$

$$N_{22}(u) = \begin{cases} \frac{1}{6}(u-5)^2 & u \in \langle 5, 7 \rangle \\ \frac{1}{3}(-2u^2 + 30u - 110) & u \in \langle 7, 8 \rangle \\ \frac{1}{6}(10-u)^2 & u \in \langle 8, 10 \rangle \\ 0 & u \notin \langle 5, 10 \rangle \end{cases}$$

$$N_{32}(u) = \begin{cases} \frac{1}{3}(u-7)^2 & u \in \langle 7, 8 \rangle \\ \frac{1}{3}(-u^2 + 18u - 79) & u \in \langle 8, 10 \rangle \\ \frac{1}{3}(11-u)^2 & u \in \langle 10, 11 \rangle \\ 0 & u \notin \langle 7, 11 \rangle \end{cases}$$

B-spline křivka je definovaná na intervalu $u \in \langle 5, 8 \rangle$

$$s(u) = \binom{1}{2} N_{02}(u) + \binom{3}{5} N_{12}(u) + \binom{6}{2} N_{22}(u) + \binom{9}{4} N_{32}(u)$$

a má dva segmenty

1. $u \in \langle 5, 7 \rangle$

$$\bar{s}(u) = \frac{1}{6} (7-u)^2 \binom{1}{2} + \frac{1}{3} (-u^2 + 12u - 34) \binom{3}{5} + \frac{1}{6} (u-5)^2 \binom{6}{2}$$

$$x(u) = -\frac{65}{3} + 8u - \frac{2}{3}u^2$$

$$y(u) = -\frac{701}{18} + \frac{133}{9}u - \frac{23}{18}u^2$$

2. $u \in \langle 7, 8 \rangle$

$$\bar{s}(u) = \frac{1}{3} (8-u)^2 \binom{3}{5} + \frac{1}{3} (-2u^2 + 30u - 110) \binom{6}{2} + \frac{1}{3} (u-7)^2 \binom{9}{4}$$

$$x(u) = -\frac{155}{3} + \frac{38}{3}u - \frac{2}{3}u^2$$

$$y(u) = -\frac{340}{9} + \frac{100}{9}u - \frac{7}{9}u^2$$

a je vykreslena z dvoch segmentov

