

Máme daný bod $X(0, -1, 3)$. Otočte ho okolo přímky dané

$$p: (2, 1, 2)^T + t \cdot (1, 1, 1)^T \quad \text{o uhol } \theta = \frac{\pi}{2}$$

Použijte quaterniony!

1) posuneme přímku a bodu, aby přímka procházela $a \in (0, 0, 0)$

$$P \mapsto P' = (0, 0, 0)^T + t(1, 1, 1)^T$$

$$X \rightarrow X' = (0, -1, 3) - (2, 1, 2) = (-2, -2, 1)$$

2) zestrojíme quaternion $q_{\theta} = \cos \frac{\pi}{4} + \bar{u} \sin \frac{\pi}{4}$, kde \bar{u} je normál.

$$\text{směr } \bar{u} = \frac{\bar{U}}{\|U\|} = \frac{(1, 1, 1)}{\sqrt{3}} = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) = \frac{\sqrt{3}}{3}(1, 1, 1)$$

$$q_{\theta} = \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3}(1, 1, 1) \cdot \frac{\sqrt{2}}{2} = q_0 + \bar{q} = \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{6}(1, 1, 1)$$

vektor (bod), který otáčíme je $\bar{v}(-2, -2, 1)$

Použijeme vztah $q_{\theta} \cdot \bar{v} \cdot q_{\theta}^*$ pro otáčení \bar{v} okolo \bar{u} s $\theta = \frac{\pi}{2}$

$$q \bar{v} q^* = \underbrace{(q_0^2 - \|\bar{q}\|^2)}_{\textcircled{1}} \bar{v} + \underbrace{2(\bar{q} \cdot \bar{v})}_{\textcircled{2}} \bar{q} + \underbrace{2q_0(\bar{q} \times \bar{v})}_{\textcircled{3}}$$

$$q_0 = \frac{\sqrt{2}}{2} \quad \bar{q} = \frac{\sqrt{6}}{6}(1, 1, 1) \quad \|\bar{q}\|^2 = \frac{6}{36} + \frac{6}{36} + \frac{6}{36} = \frac{18}{36} = \frac{1}{2} \quad q_0^2 = \frac{1}{2}$$

$$\textcircled{1} \quad \left(\frac{1}{2} - \frac{1}{2}\right) \bar{v} = \boxed{(0, 0, 0)}$$

$$\textcircled{2} \quad \bar{q} \cdot \bar{v} = \frac{\sqrt{6}}{6}(1, 1, 1) \cdot (-2, -2, 1) = \frac{\sqrt{6}}{6} \cdot (-2) = \frac{-\sqrt{6}}{2}$$

$$2(\bar{q} \cdot \bar{v}) \cdot \bar{q} = -\sqrt{6} \left(\frac{\sqrt{6}}{6}\right)(1, 1, 1) = \boxed{(-1, -1, -1)}$$

$$\textcircled{3} \quad 2 \cdot \frac{\sqrt{2}}{2} (\bar{q} \times \bar{v}) = \sqrt{2} \cdot \frac{\sqrt{6}}{2}(1, -1, 0) = \boxed{\sqrt{3}(1, -1, 0)}$$

$$\bar{q} \times \bar{v} = \frac{\sqrt{6}}{6} \cdot \begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ -2 & -2 & 1 \end{vmatrix} = \frac{\sqrt{6}}{6} (x - 2z - 2y + 2z + 2x - y) = \frac{\sqrt{6}}{6} (3, -3, 0) = \frac{\sqrt{6}}{6} (1, -1, 0) = \frac{\sqrt{6}}{2} (1, -1, 0)$$

$$\textcircled{1} + \textcircled{2} + \textcircled{3} \quad (0, 0, 0) + (-1, -1, -1) + \sqrt{3}(1, -1, 0) = (\sqrt{3} - 1, -\sqrt{3} - 1, -1)$$

$$\text{Přidáme naopak } + (2, 1, 2) \Rightarrow \underline{\underline{(\sqrt{3} + 1, -\sqrt{3}, 1)}}$$