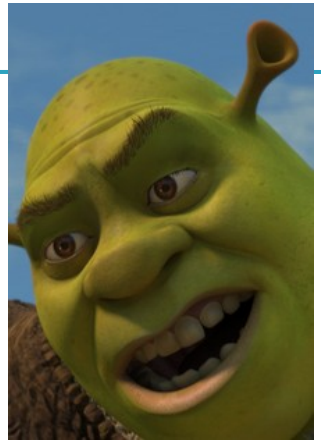
Computer Graphics III – Approximate global illumination computation

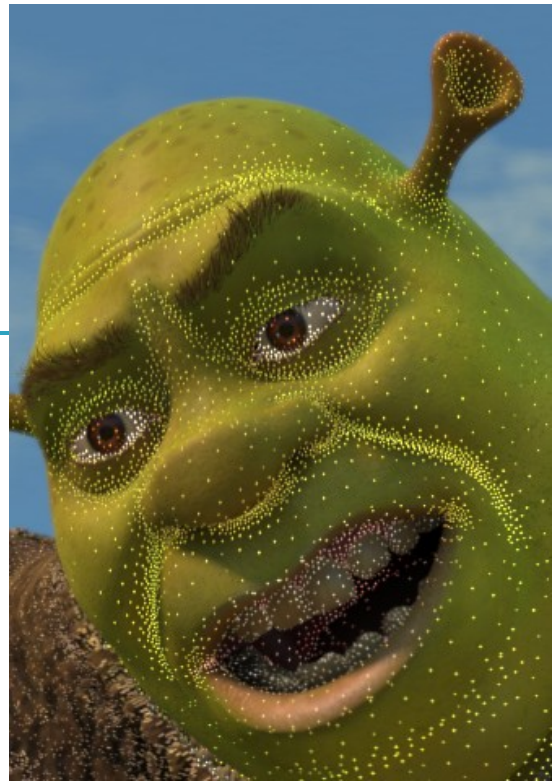
Jaroslav Křivánek, MFF UK
Jaroslav.Krivanek@mff.cuni.cz

Approximate GI methods

Irradiance caching

Jaroslav Křivánek
Charles University, Prague
Jaroslav.Krivanek@mff.cuni.cz





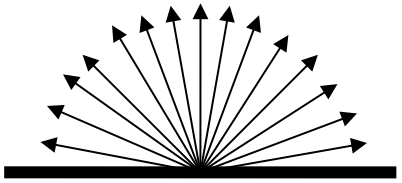
Motivation

- Spatial coherence
  - Diffuse indirect illumination changes slowly over surfaces

Indirect irradiance – changes slowly

Irradiance caching

- Sparse locations for full DRT computation
- Resulting irradiance stored in a cache
- Most pixels interpolated from cached records



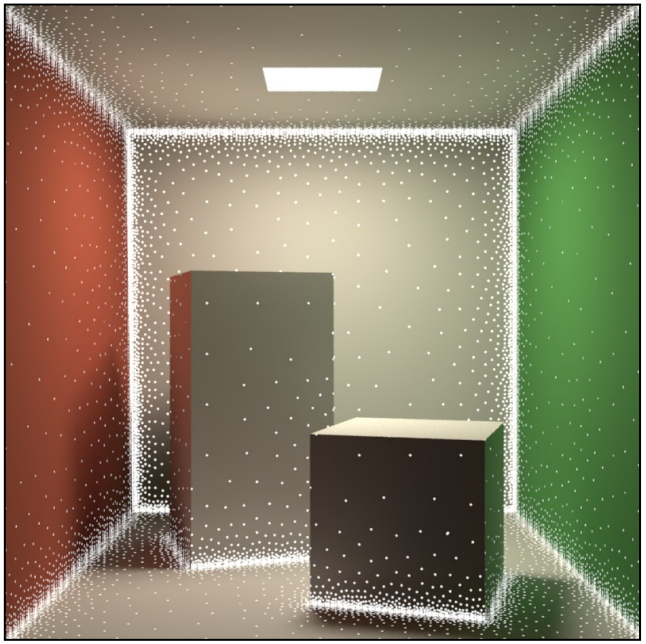


Image credit: Okan Arikan

Irradiance caching

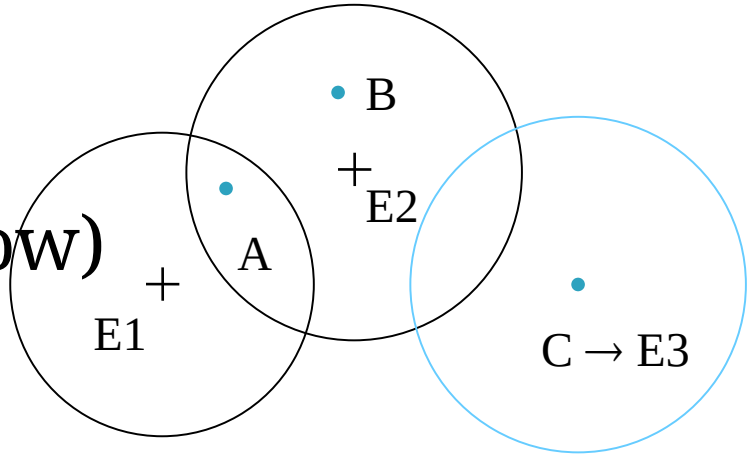
- Faster computation of the *diffuse component* of indirect illumination
- Diffuse reflection

$$L_o(\mathbf{p}) = E(\mathbf{p}) * \rho_d(\mathbf{p}) / \pi$$

- View-independence
  - Outgoing radiance independent of view direction
  - Total irradiance is all we need => cache irradiance

Irradiance caching

- Lazy evaluation of new irradiance values
  - Only if cannot be interpolated from existing ones
- Example: Values E1 and E2 already stored
  - Interpolate at A (fast)
  - Extrapolate at B (fast)
  - Add new record at C (slow)

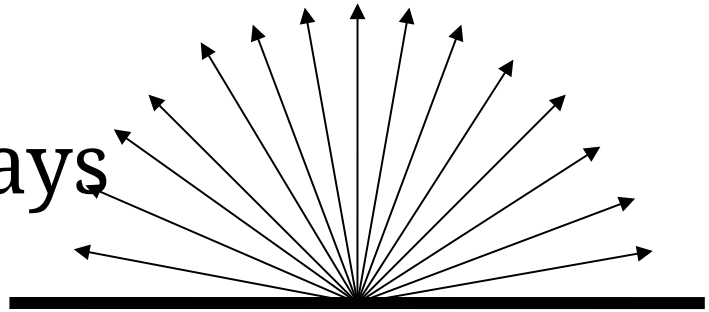


Irradiance caching pseudocode

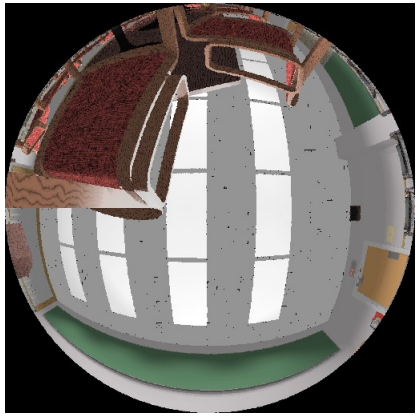
```
GetIrradiance(p):
   Color E = InterpolateFromCache(p);
   if( E == invalid )
      E = SampleHemisphere(p);
      InsertIntoCache(E, p);
   return E;
```

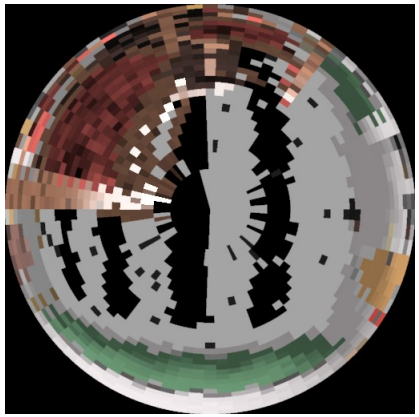
Indirect irradiance calculation

```
E = SampleHemisphere(p);
```



- Cast 500-5000 secondary rays (user-specified)
- Compute illumination at intersection
  - Direct illumination only, or
  - Path tracing, or
  - Photon map radiance estimate, or
  - Query in (another) irradiance cache
  - No emission taken into account!

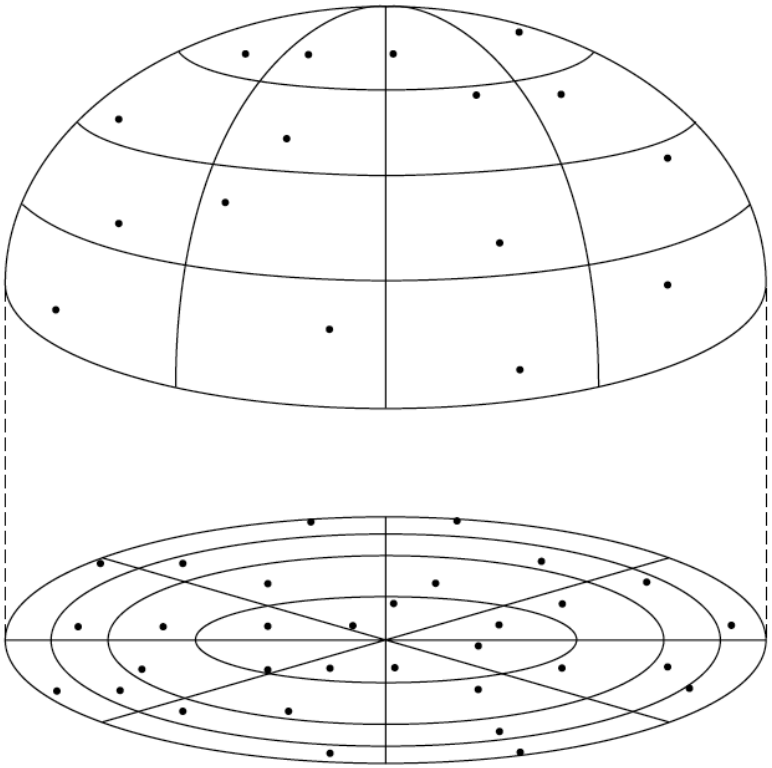




Indirect irradiance calculation

```
E = SampleHemisphere(p);
```

- Stratified Monte Carlo hemisphere sampling
  - Subdivide hemisphere into cells
  - Choose a random direction in each cell and trace ray



Indirect irradiance calculation

```
E = SampleHemisphere(p);
```

- Estimating irradiance at **p**:

$$E(\mathbf{p}) = \int L_i(\mathbf{p}, \omega_i) \cos\theta_i \, \mathrm{d}\omega_i$$

- General form of the stratified estimator

$$E(\mathbf{p}) \approx \frac{1}{MN} \sum_{j=0}^{M-1} \sum_{k=0}^{N-1} \frac{f(\theta_{j,k}, \phi_{j,k})}{p(\theta_{j,k}, \phi_{j,k})}$$

Indirect irradiance calculation

```
E = SampleHemisphere(p);
```

- For irradiance calculation, the integrand is:

$$L(\theta, \phi)\ \cos\theta$$

- PDF:

$$p(\theta, \phi) = \frac{\cos\theta}{\pi}$$

Indirect irradiance calculation

- Irradiance estimator for IC:

$$E(\mathbf{p}) \approx \frac{\pi}{MN} \sum_{j=0}^{M-1} \sum_{k=0}^{N-1} L_{j,k}$$

- $L_{j,k}$ … radiance sample from direction:

$$(\theta_{j,k}, \phi_{j,k}) = \left(\arccos \sqrt{1 - \frac{j + \zeta_{j,k}^1}{M}}, 2\pi \frac{k + \zeta_{j,k}^2}{N} \right)$$

- M, N … number of divisions along q and f
- $\zeta_{j,k}^1, \zeta_{j,k}^2$ … random numbers from R(0,1)

Irradiance caching pseudocode

```
GetIrradiance(p):
    Color E = InterpolateFromCache(p);
    if( E == invalid )
        E = SampleHemisphere(p);
        InsertIntoCache(E, p);
    return E;
```

Record spacing

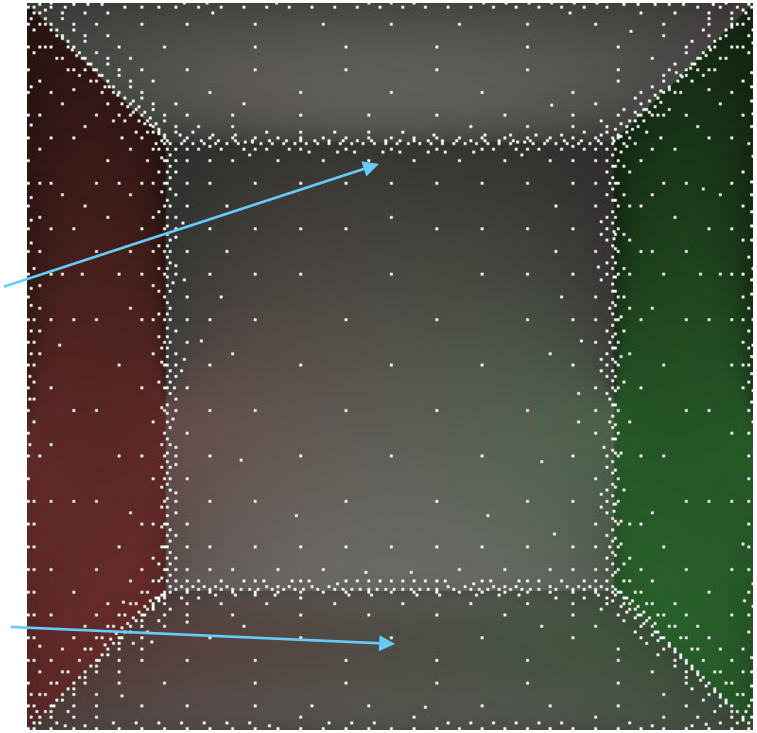
- If E(**p**) changes slowly => interpolate more
- If E(**p**) changes quickly => interpolate less

- What is the upper bound on rate of change (i.e. gradient) of irradiance?
- Answer from the “worst case” analysis (omitted)

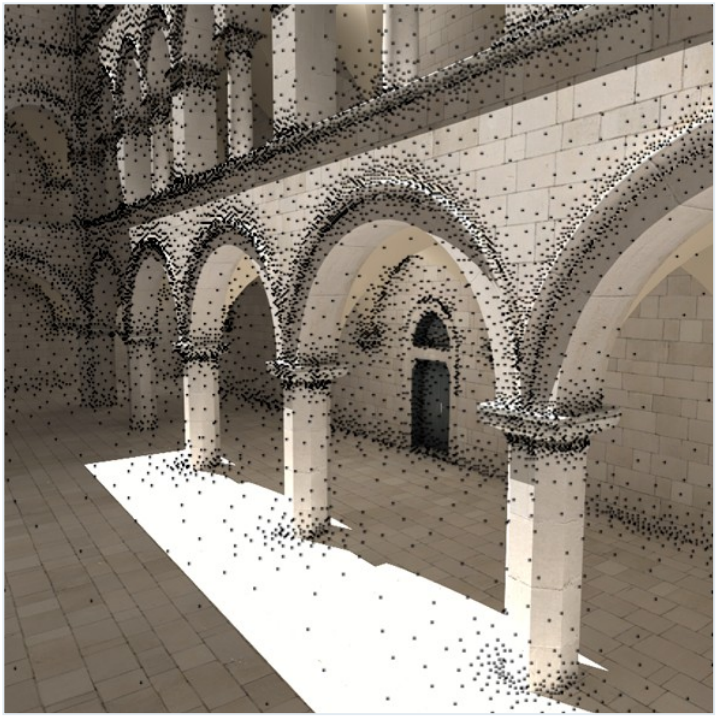
Record spacing

- Near geometry
  → dense spacing
  - Geometry = source of indirect illumination

- Open spaces
  → sparse sampling



Record spacing



Irradiance interpolation

```
E = InterpolateFromCache(p)
```

- Weighted average:

$$E(\mathbf{p}) = \frac{\sum_{i \in S(\mathbf{p})} E_i(\mathbf{p}) w_i(\mathbf{p})}{\sum_{i \in S(\mathbf{p})} w_i(\mathbf{p})},$$

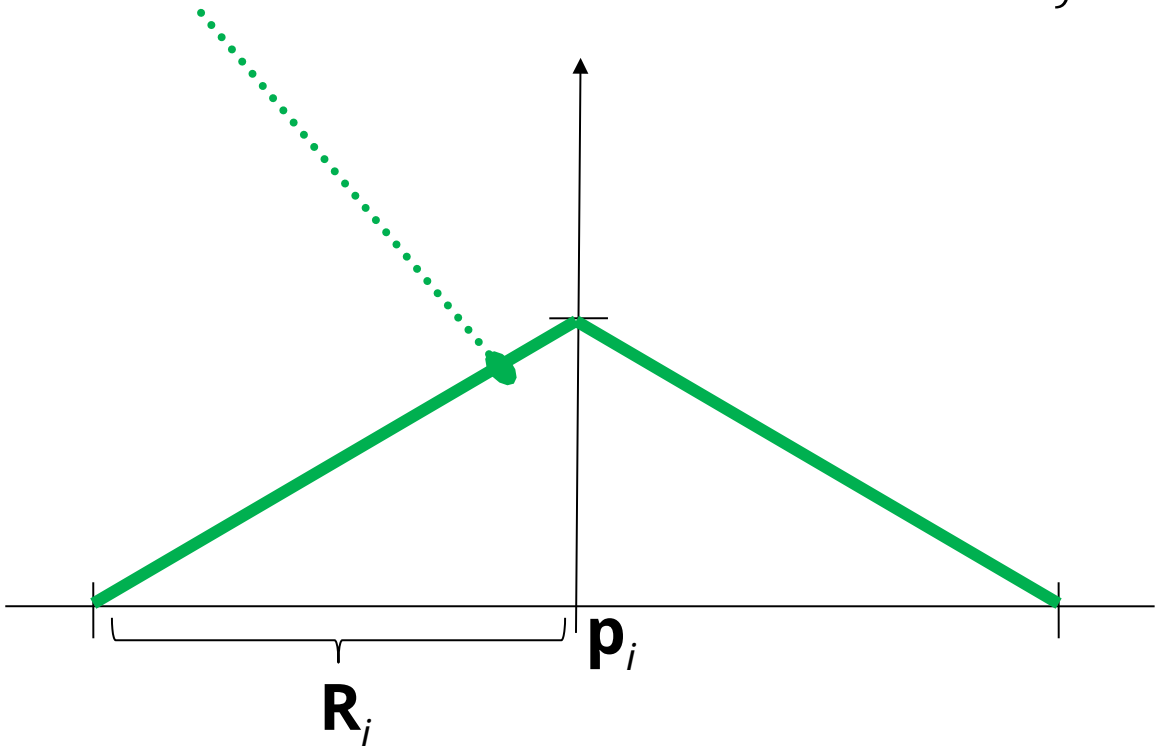
- Records used for interpolation:

$$S(\mathbf{p}) = \{i;\ w_i(\mathbf{p}) > 0\}$$

Weighting function

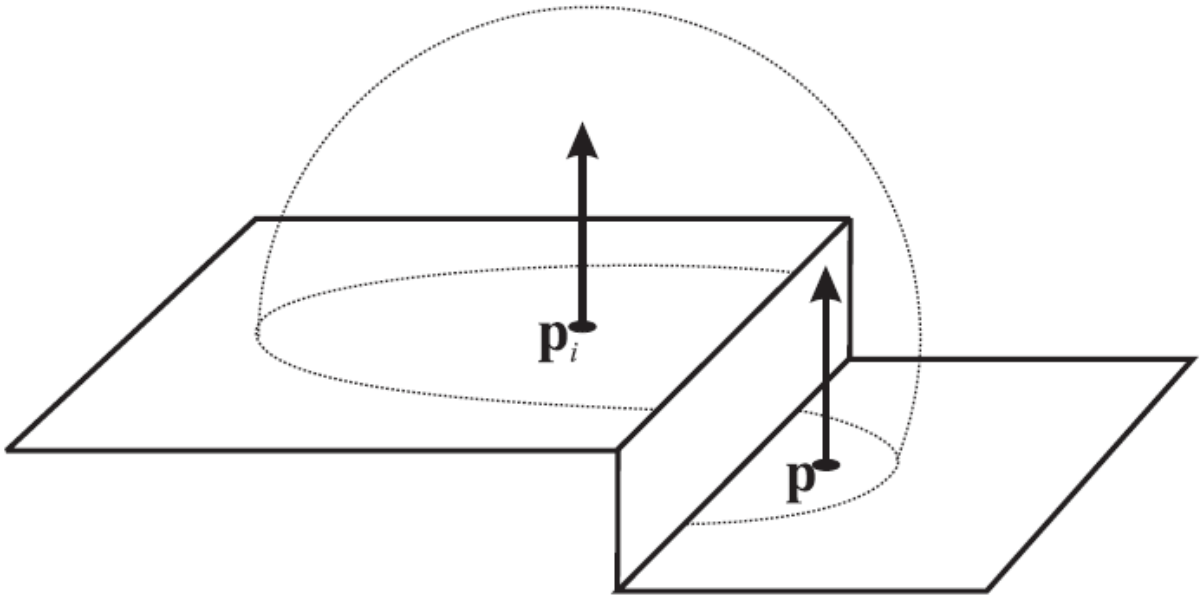
[Tablellion and Lamorlette 04]

$$w_i(\mathbf{p}) = 1 - \kappa \max\left\{ \frac{\|\mathbf{p} - \mathbf{p}_i\|}{\mathrm{clamp}(2R_i, R_{\min}, R_{\max})}, \frac{\sqrt{1 - \mathbf{n} \cdot \mathbf{n}_i}}{\sqrt{1 - \cos 10^\circ}} \right\}$$



Heuristic “behind” test

- Record at p_i rejected from interpolation at p if p is “behind” p_i



Irradiance caching pseudocode

```
GetIrradiance(p):
   Color E = InterpolateFromCache(p);
   if( E == invalid )
      E = SampleHemisphere(p);
      InsertIntoCache(E, p);
   return E;
```

Irradiance cache record

```
InsertIntoCache(E, p);
```

• Vector3 position	Position in space
• Vector3 normal	Normal at `position’
• float R	Validity radius
• Color E	Stored irradiance
• Color dEdP[3]	Gradient w.r.t. translation
• Color dEdN[3]	Gradient w.r.t. rotation

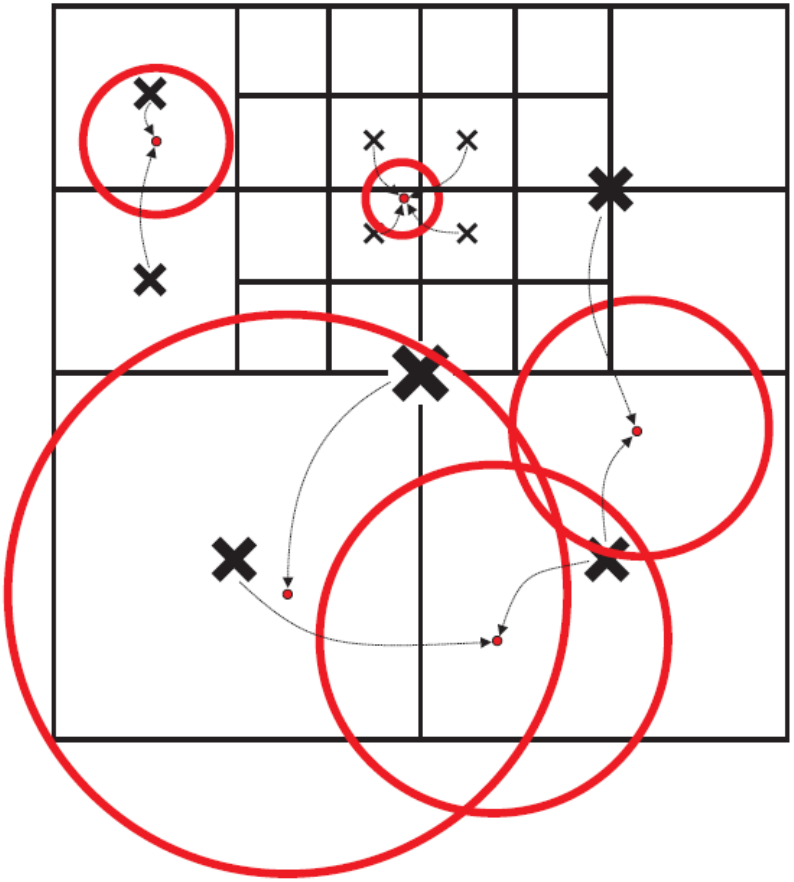
Irradiance cache data structure

```
InsertIntoCache(E, p);
```

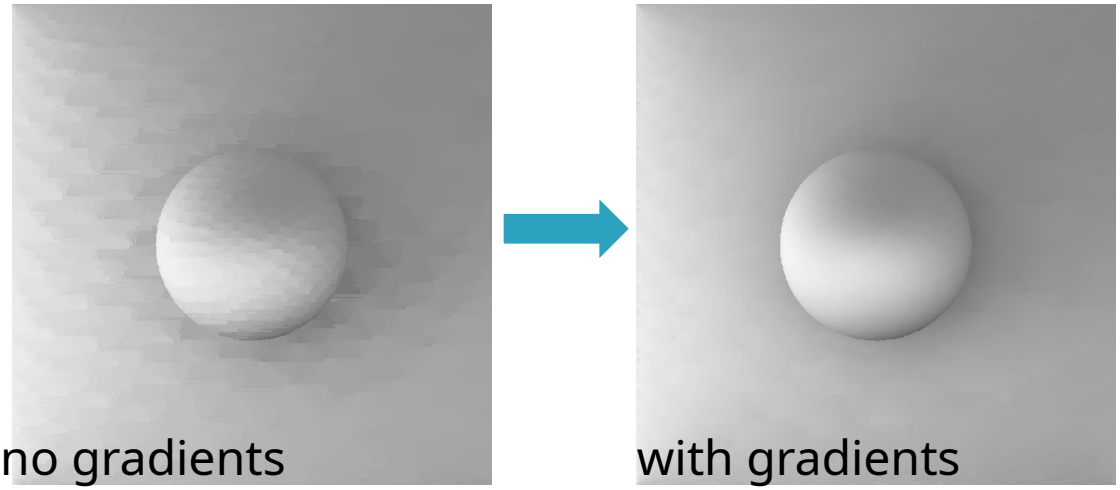
- Requirements
  - Fast incremental updates (records stored on the fly)
  - Fast query for all records (spheres) overlapping a given point p

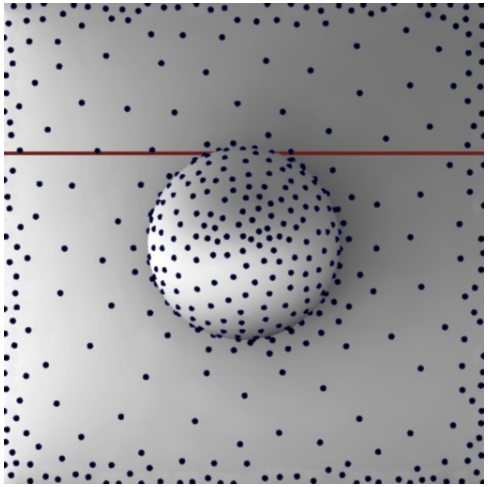
Data structure: Octree

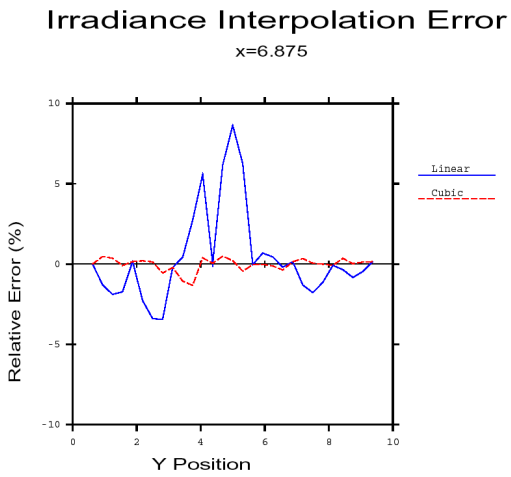
```
InsertIntoCache(E, p);
```



Irradiance gradients







Irradiance gradients

- Essential for smooth interpolation
- Calculated during hemisphere sampling
  - i.e. no extra rays, little overhead
- Stored as a part of the record in the cache
- Used in interpolation

Rotation gradient

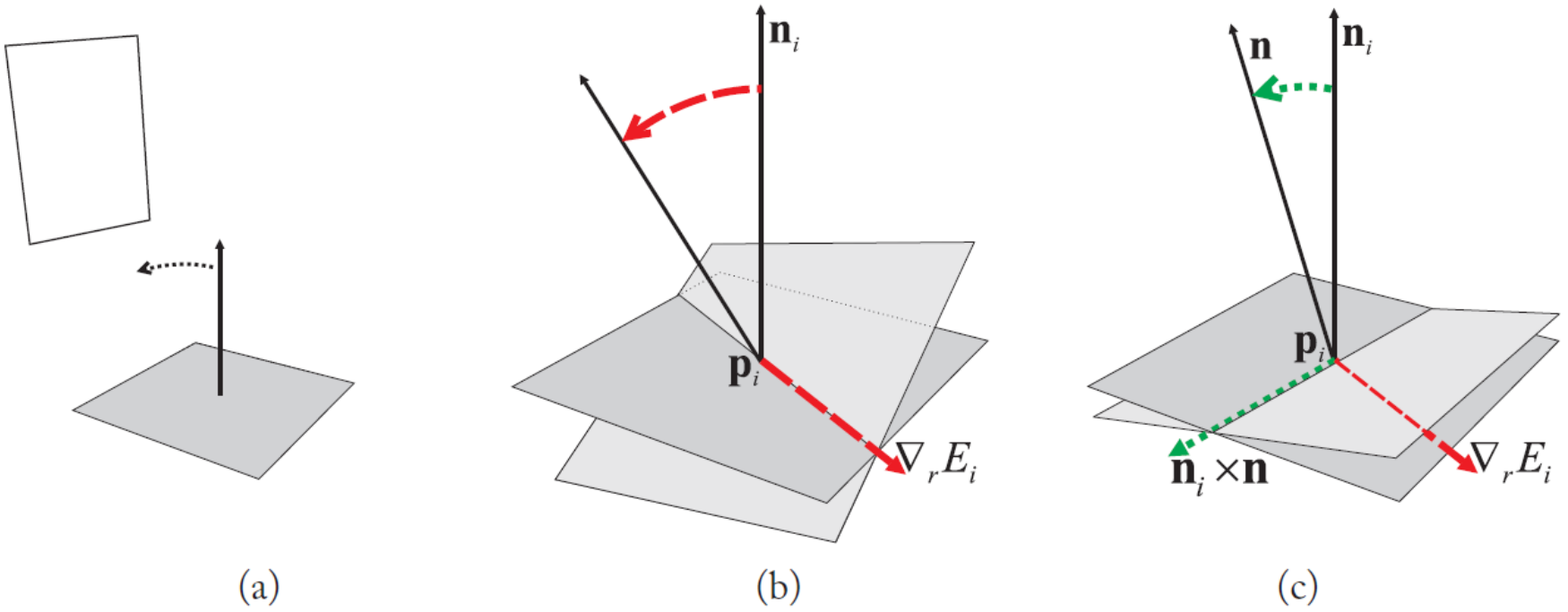
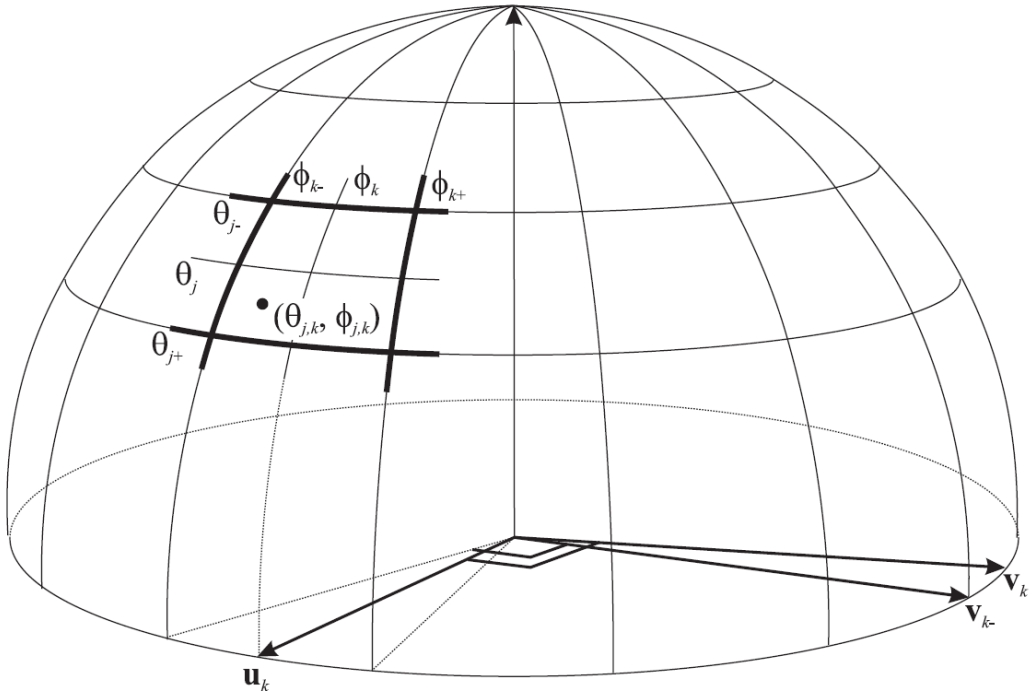


Figure 2.4: (a) As the surface element is rotated towards the bright surface, irradiance increases. (b) The rotation gradient $\nabla_r E_i$ of cache record i gives the axis of rotation that produces maximum increase in irradiance. The gradient magnitude is the irradiance derivative with rotation around that axis. (c) When the surface element is rotated around any arbitrary axis (in our example determined by the change in surface normal as $\mathbf{n}_i \times \mathbf{n}$) the irradiance derivative is given by the dot product of the axis of rotation and the rotation gradient: $(\mathbf{n}_i \times \mathbf{n}) \cdot \nabla_r E_i$.

Rotation gradient formula

$$\nabla_r E \approx \frac{\pi}{MN} \sum_{k=0}^{N-1} \left(\mathbf{v}_k \sum_{j=0}^{M-1} - \tan\theta_j \cdot L_{j,k} \right)$$



Translation gradient

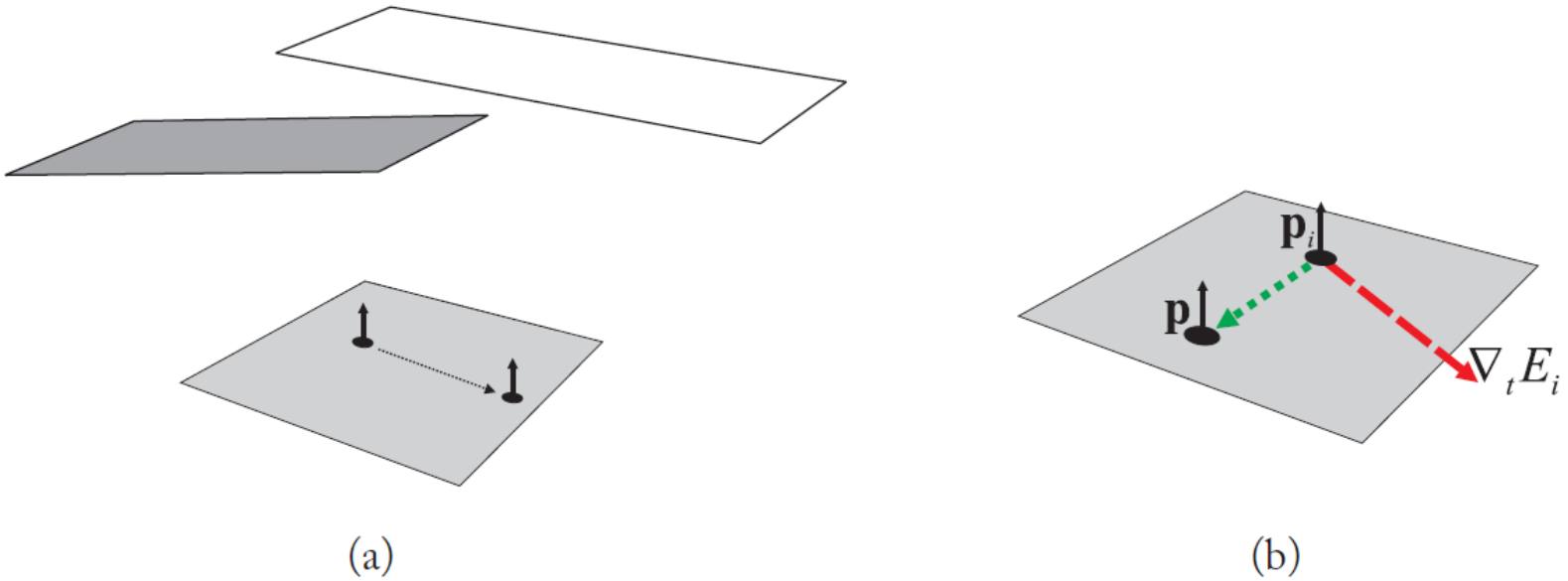
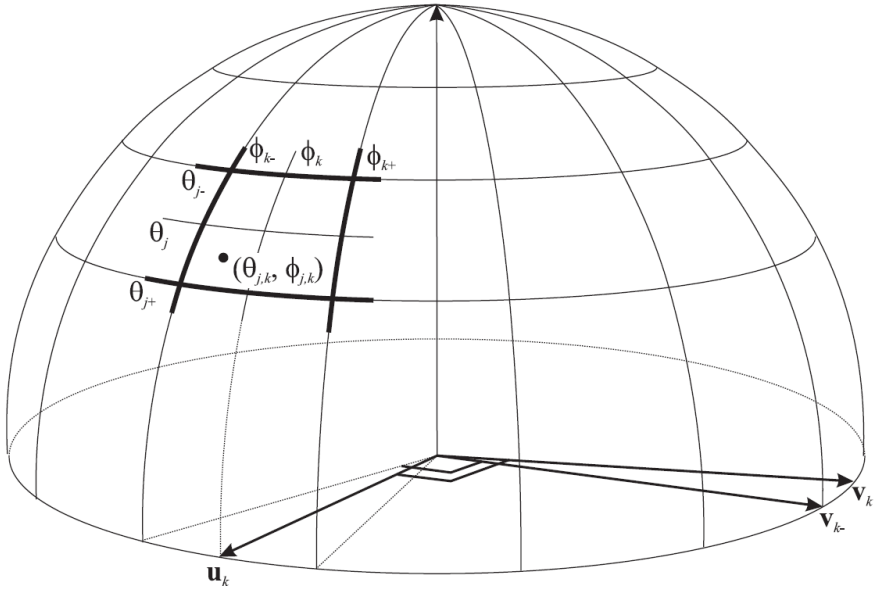


Figure 2.6: (a) As the surface element is translated, it becomes more exposed to the bright surface, and irradiance increases. (b) The translation gradient $\nabla_t E_i$ of record i gives the direction of translation that produces the maximum increase in irradiance. The gradient magnitude is the irradiance derivative with respect to translation along that direction. When a surface element is translated along any arbitrary direction, a first-order approximation of the change in irradiance is given by the dot product of the translation vector and the translation gradient: $(\mathbf{p} - \mathbf{p}_i) \cdot \nabla_t E_i$.

Translation gradient formula

$$\nabla_t E \approx \sum_{k=0}^{N-1} \Bigg[\mathbf{u}_k \frac{2\pi}{N} \sum_{j=1}^{M-1} \frac{\cos^2\theta_{j\_} \sin\theta_{j\_}}{\min\{r_{j,k}, r_{j-1,k}\}} (L_{j,k} - L_{j-1,k}) + \mathbf{v}_{k\_} \sum_{j=0}^{M-1} \frac{\cos\theta_j (\cos\theta_{j\_} - \cos\theta_{j+})}{\sin\theta_{j,k} \min\{r_{j,k}, r_{j,k-1}\}} (L_{j,k} - L_{j,k-1}) \Bigg]$$



Irradiance interpolation w/ grads

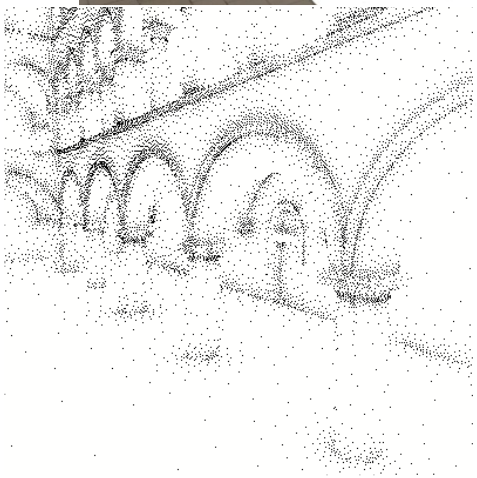
```
E = InterpolateFromCache(p)
```

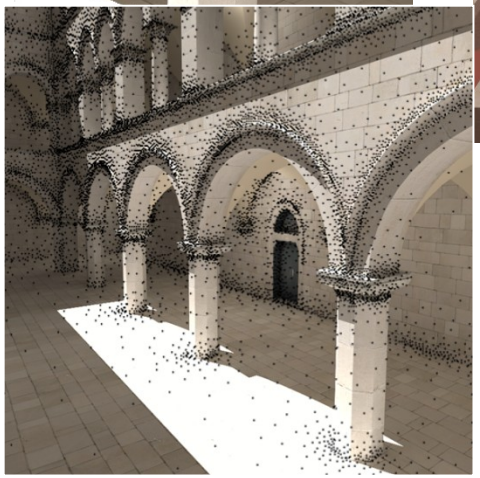
- Weighted average:

$$E(\mathbf{p}) = \frac{\sum_{i \in S(\mathbf{p})} E_i(\mathbf{p}) w_i(\mathbf{p})}{\sum_{i \in S(\mathbf{p})} w_i(\mathbf{p})},$$

$$E_i(\mathbf{p}) = E_i + (\mathbf{n}_i \times \mathbf{n}) \cdot \nabla_r E_i + (\mathbf{p} - \mathbf{p}_i) \cdot \nabla_t E_i$$

Irradiance caching examples





Irradiance caching examples

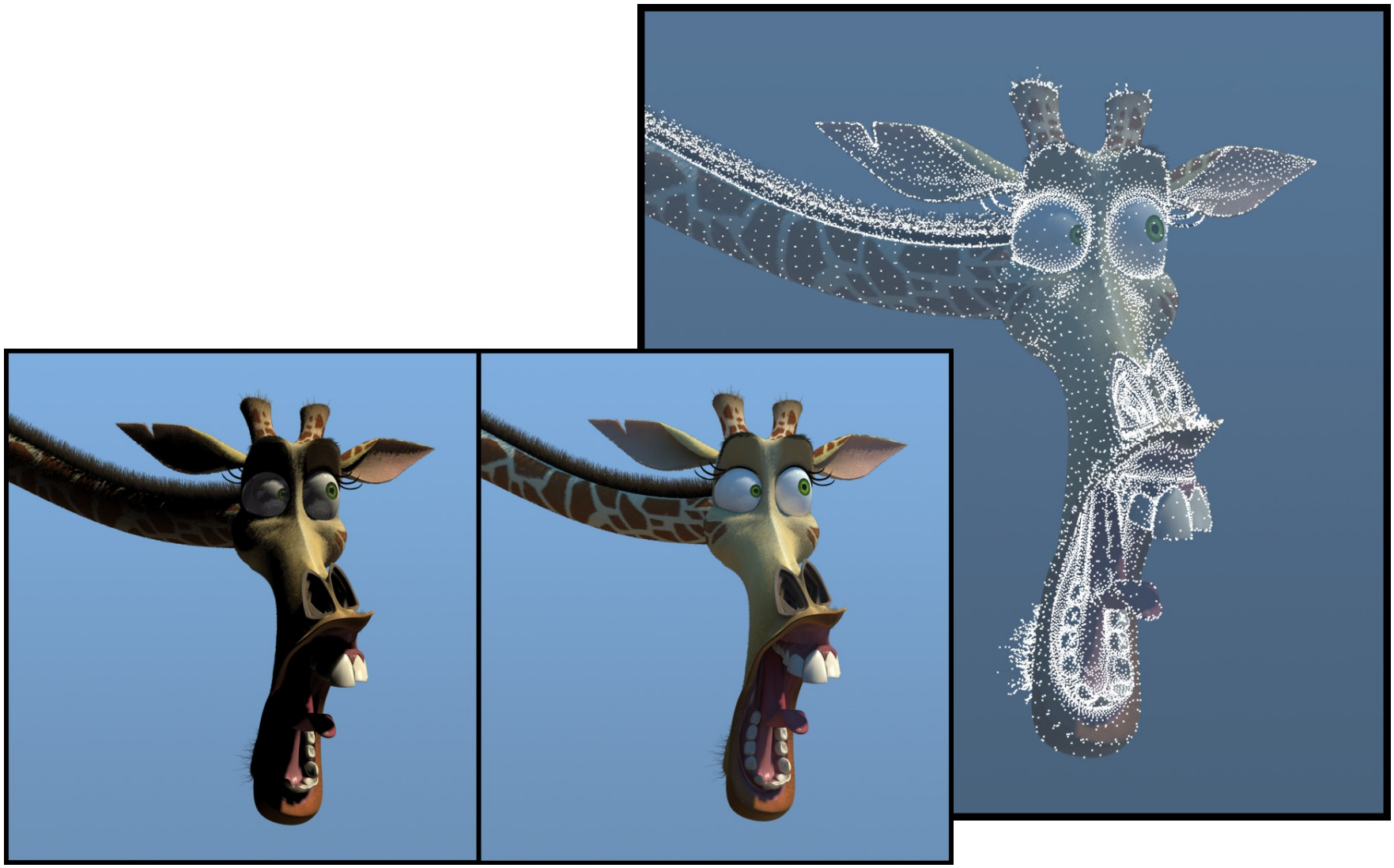


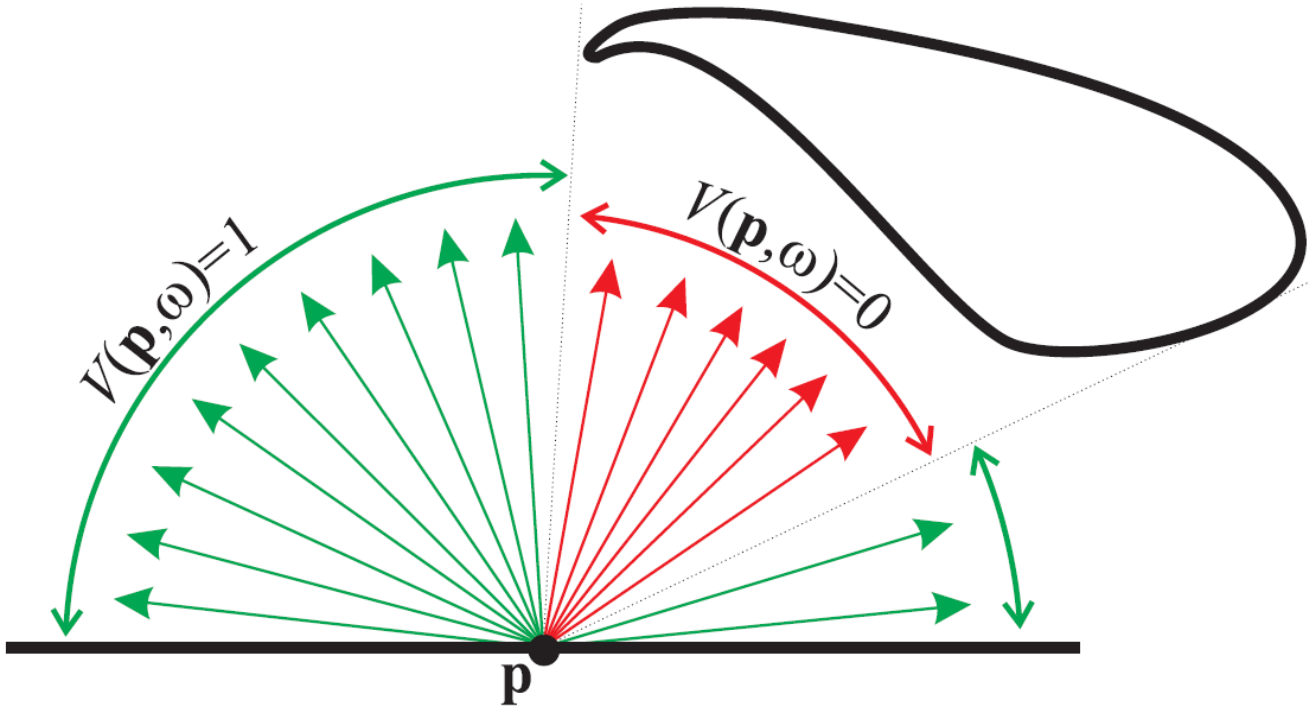
Image credit: Eric Tabellion, PDI DreamWorks

Irradiance caching examples

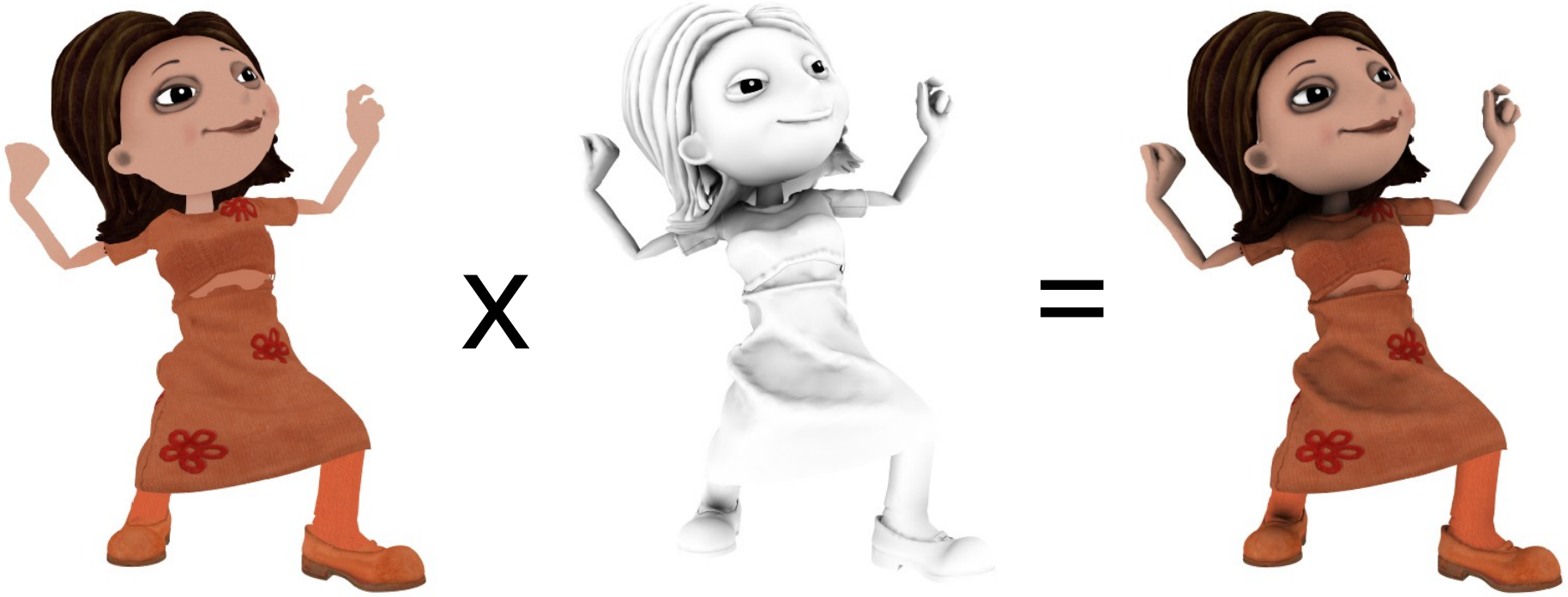
Image credit: Eric Tabellion, PDI DreamWorks

Ambient occlusion

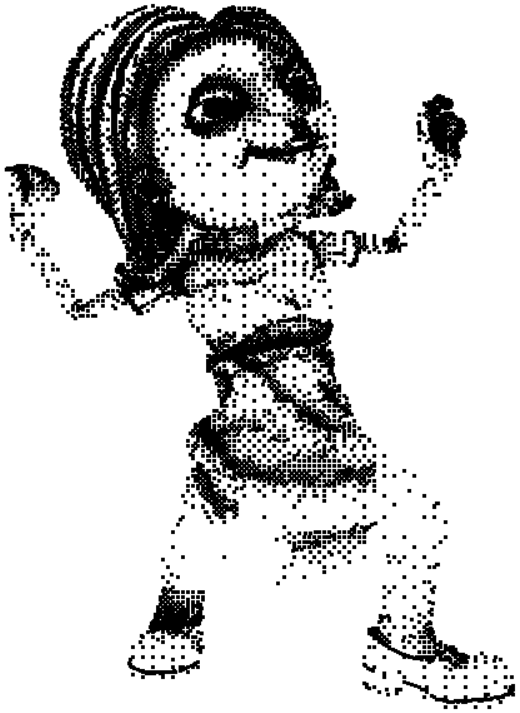
$$A(\mathbf{p}) = \frac{1}{\pi} \int_{H^+} V(\mathbf{p}, \omega) \cos\theta \, \mathrm{d}\omega$$



Ambient occlusion



Ambient occlusion caching



Conclusion

- Fast indirect illumination of diffuse surfaces
  - Sparse sampling & fast interpolation

- Biased
- Not consistent