Computer graphics III – Bidirectional path tracing

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50 60
Jerome White

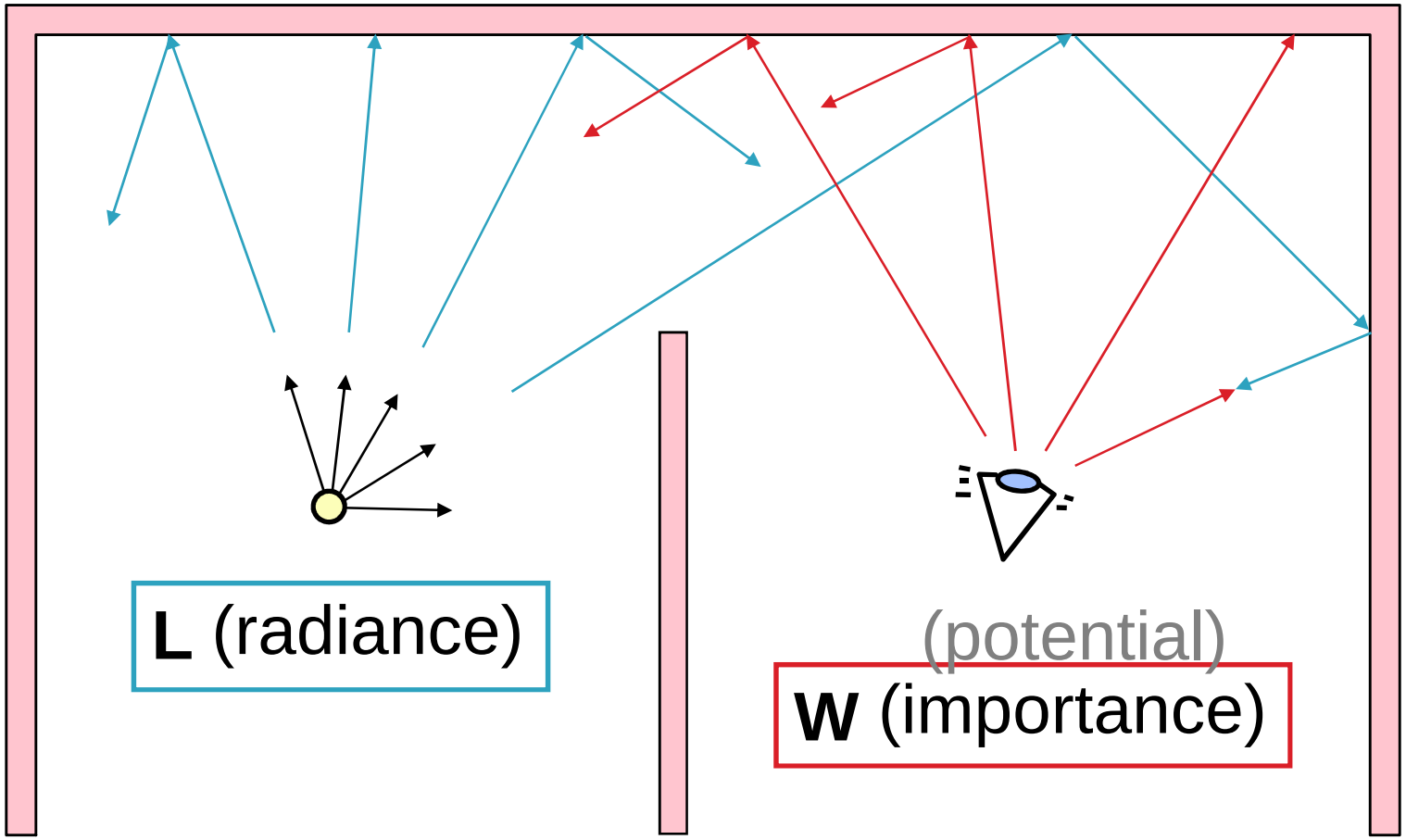

corona
Martin Geupel (DeadClown)

Measurement equation

Measurement equation

- Rendering equation enables evaluating radiance at isolated points in the scene
- But in fact, we are interested in **average radiance** over a pixel: an **integral**, again?!
- Yes, it's called the **Measurement (potential) equation**

Transport of radiance and visual importance



Visual importance (potential)

- W_e describes how important is the incident radiance to the sensor response
- One step into the scene: Incident radiance on the sensor = outgoing radiance from other scene points
- And we can go on to 2, 3, … steps into the scene…

- As a result, W_e can be interpreted as an (imaginary) transport quantity emitted from the sensor (similarly to how radiance L_e is emitted from light sources)
- In this interpretation, we call W_e the **emitted importance function**

Transport of visual importance: Potential equation

- The importance function is transported by the similar rules to radiance and settles down on an **equilibrium (steady state)** given by the **equilibrium visual importance function *W***:

$$W(\mathbf{x}, \omega_o) = W_e(\mathbf{x}, \omega_o) + \int_{H(\mathbf{x})} W(\mathrm{r}(\mathbf{x}, \omega_i), -\omega_i) \cdot f_r(\mathbf{x}, \omega_o \rightarrow \omega_i) \cdot \cos\theta_i \, \mathrm{d}\omega_i$$

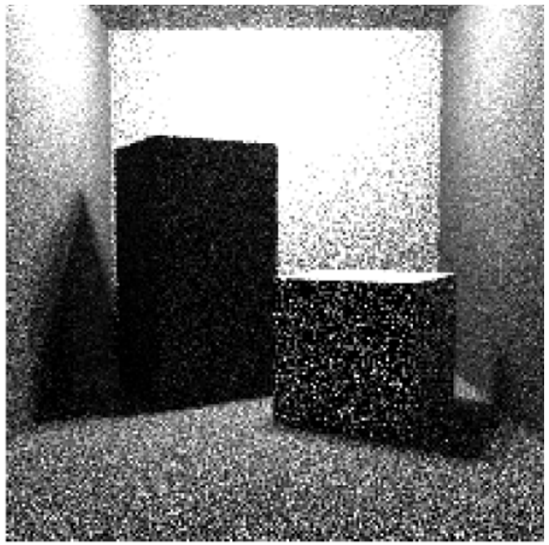
As in the rendering equation except that the BRDF arguments are exchanged (No difference for reflection because the BRDF is symmetrical, but it makes difference for transmission, which is in general not symmetrical.)

Duality of importance and radiance

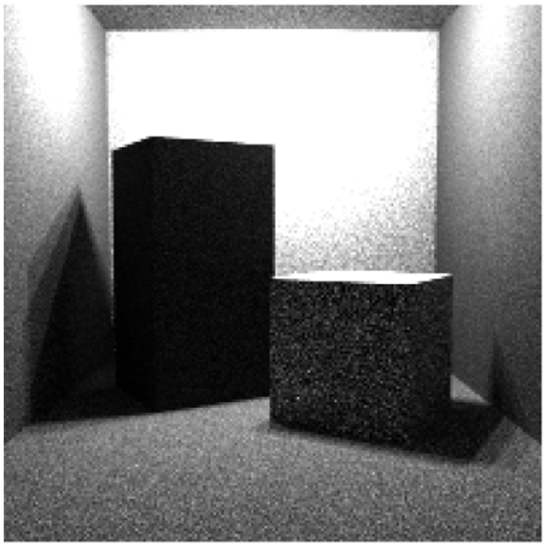
- Path tracing recursively solves the rendering equation
- Similarly, **light tracing** recursively solves the importance transport equation
  - Light paths start at the light sources and are traced into the scene using exactly the same rules as photons in photon mapping
  - They may either hit the sensor by chance (for a finite aperture camera) or we can explicitly connect vertices to the sensor (as in explicit light source sampling in PT)

Light tracing

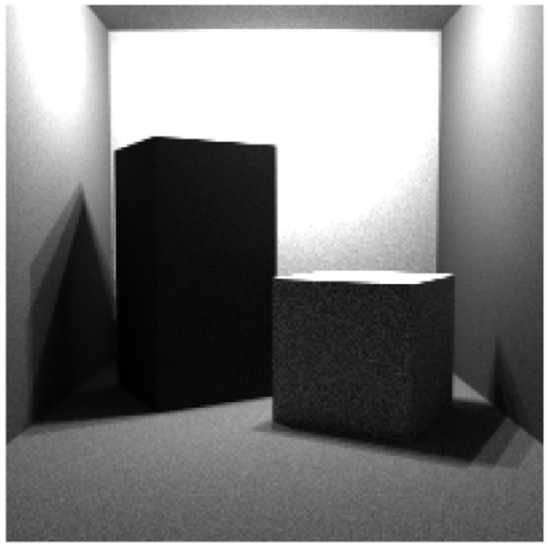
100,000 light rays



1,000,000 light rays



10,000,000 light rays



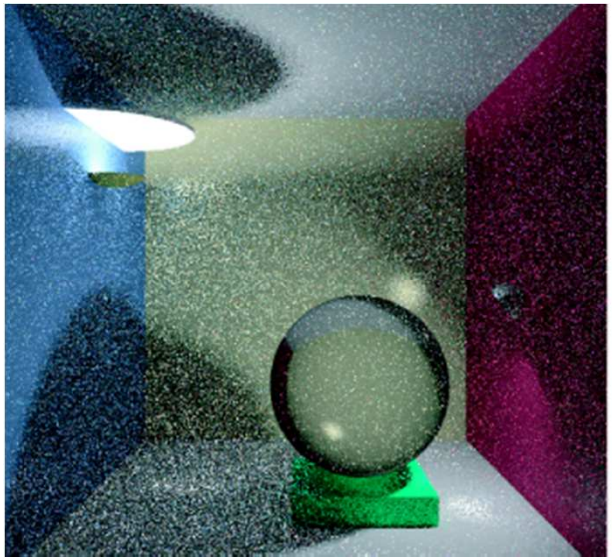
100,000,000 light rays

Image: Dutre et al. Advanced Global Illumination

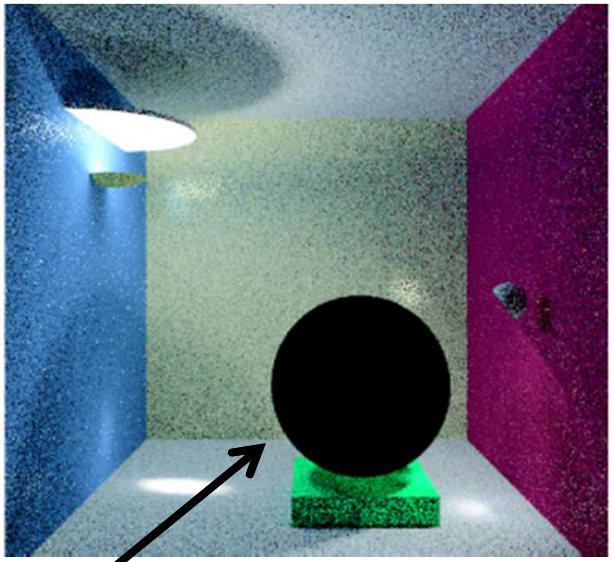
Light tracing in practice

- Generally less efficient than PT
- But it certain case, it may be much better. One example are **caustics.**
- Light tracing and path tracing are the basis of bidirectional methods, such as
  - Bidirectional path tracing, BPT
  - Photon mapping, etc.

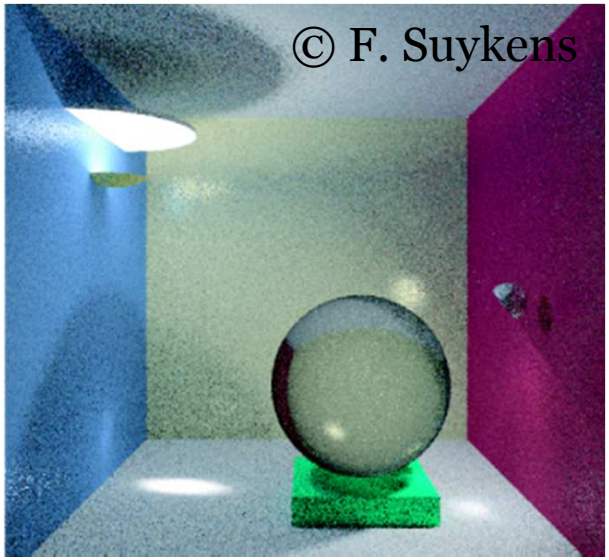
Comparison



Path tracing



Light tracing



Bidirectional path tracing

Q: Why is the glass sphere entirely black?

Advanced light transport simulation methods

Bidirectional path tracing (BPT) vs. (unidirectional) path tracing (PT)

BPT, 25 path per pixel

PT, 56 path per pixel

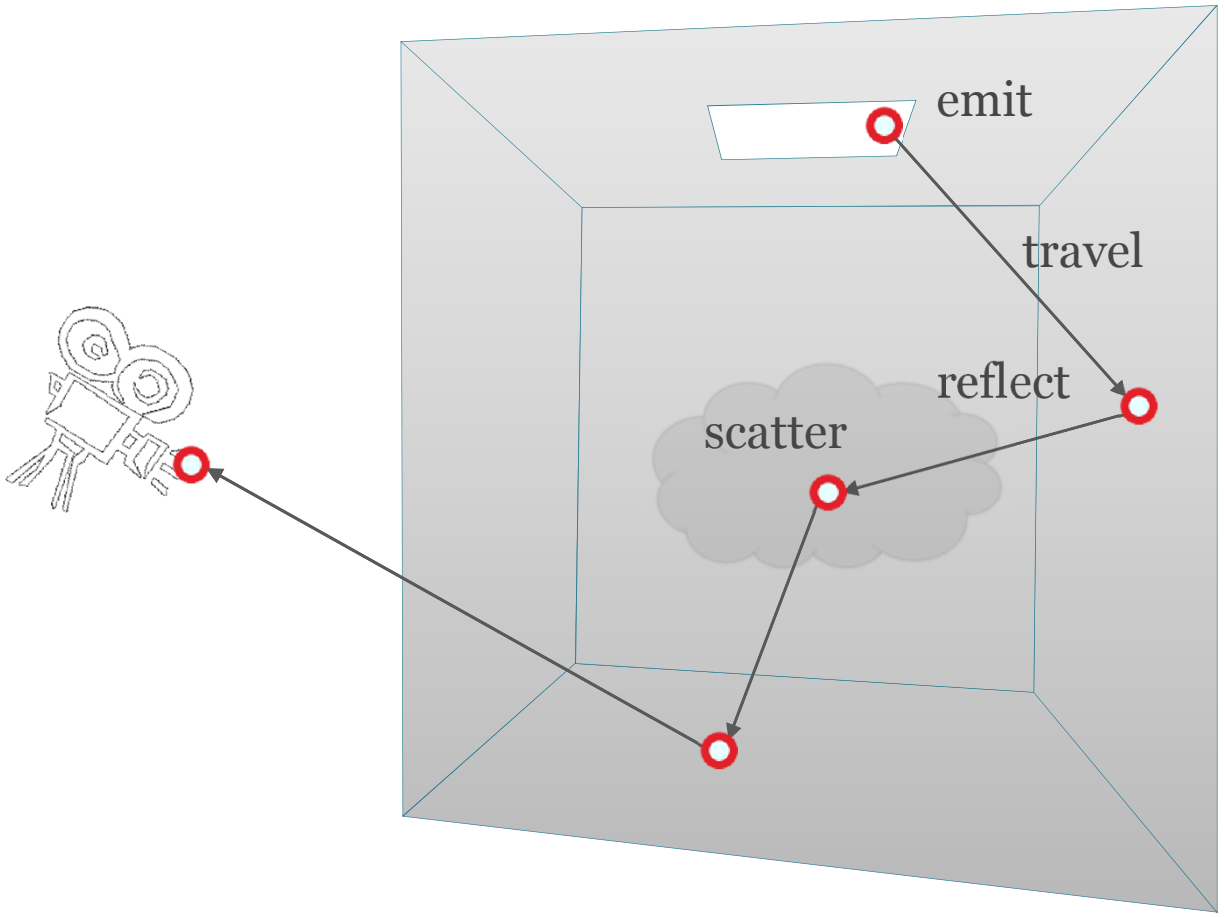
Image: Eric Veach

Path integral formulation of light transport

Light transport expressed as an integral over the space o light transport paths

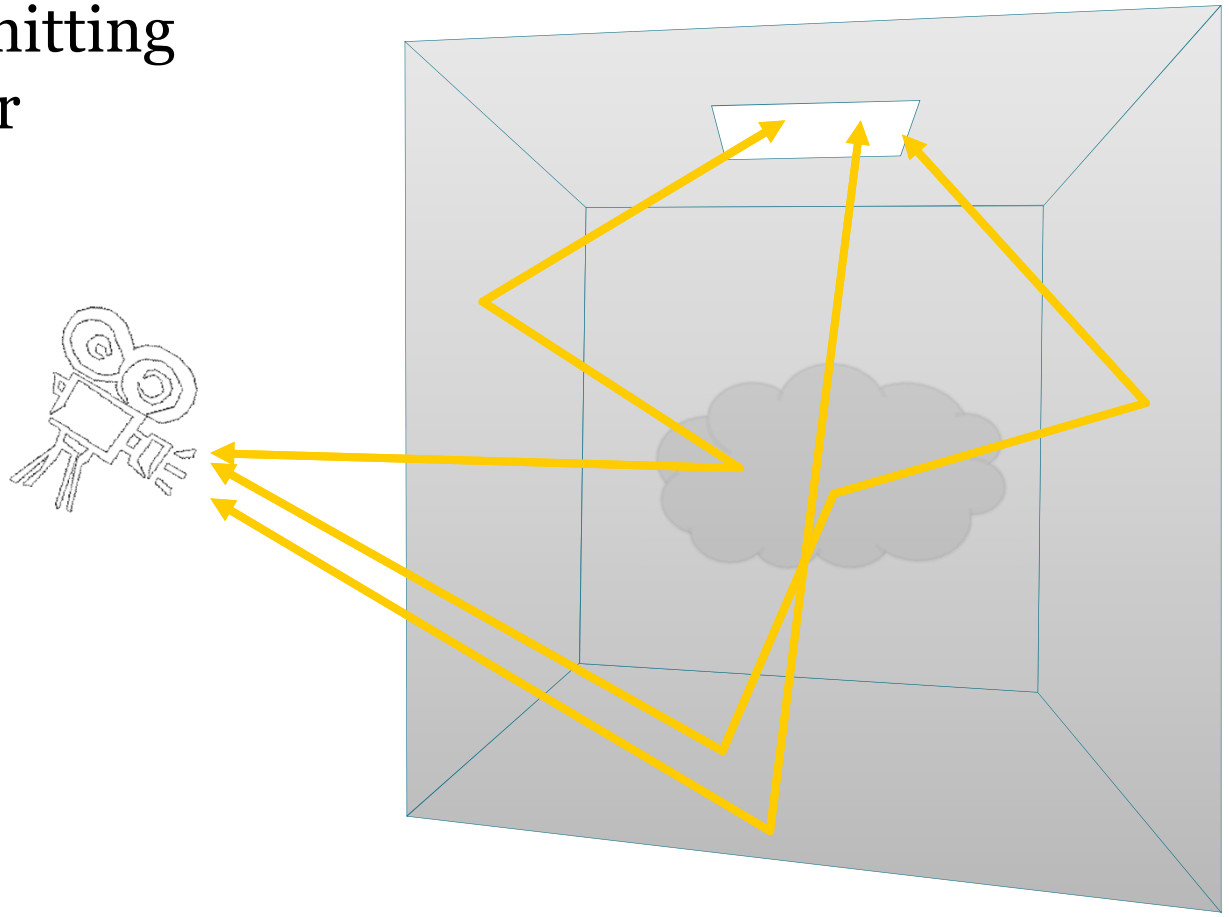
Light transport

- Geometric optics



Light transport

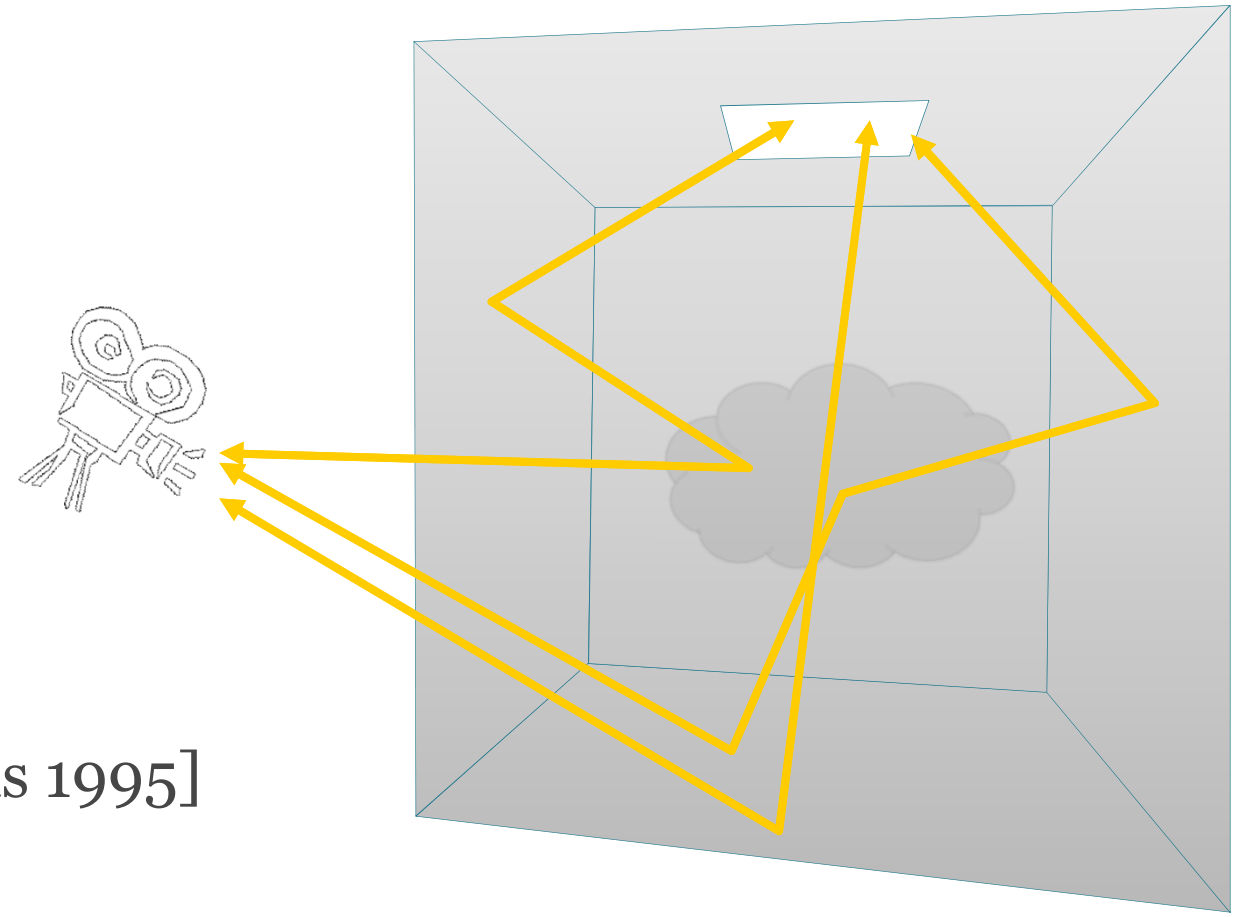
- **Camera response**
  - all paths hitting the sensor



Path integral formulation

$$I_j = \int_\Omega f_j(\bar{x})\ \mathrm{d}\mu(\bar{x})$$

- I_j: camera resp. (j-th pixel value)
- $\int_\Omega$: all paths
- $f_j(\bar{x})$: measurement contribution function



[Veach and Guibas 1995]
[Veach 1997]

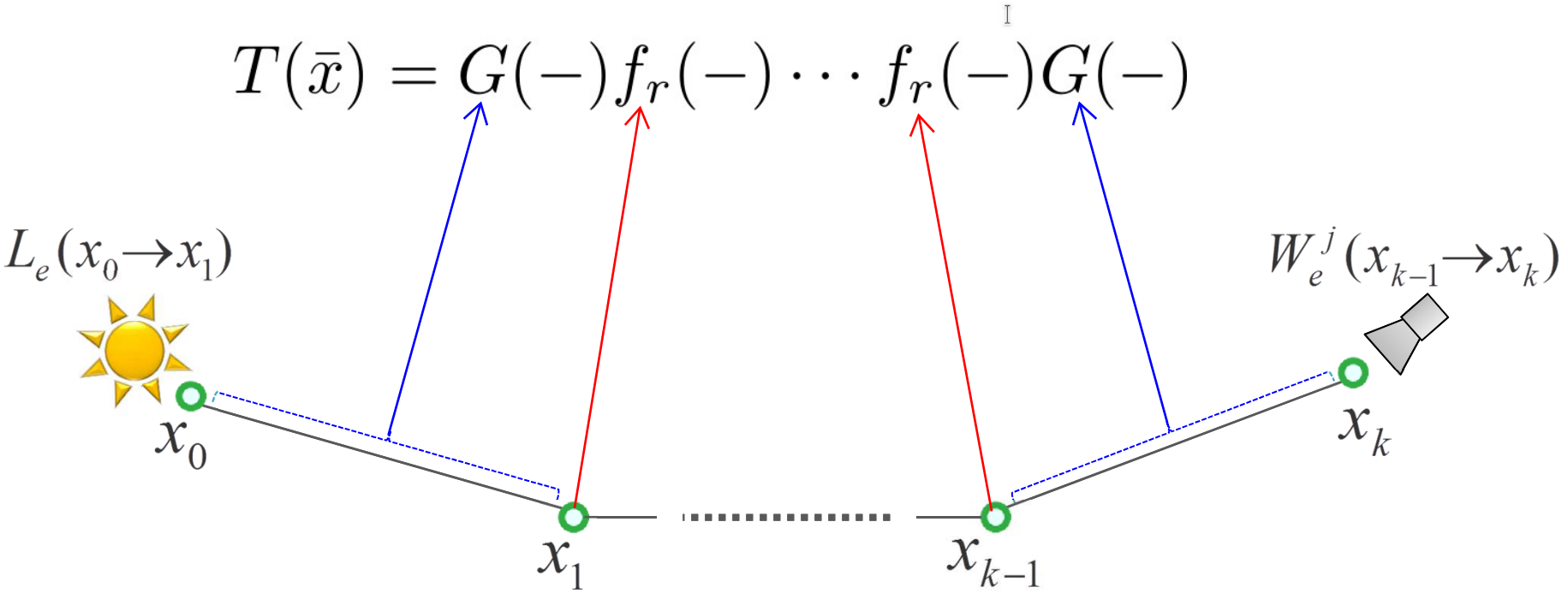
Measurement contribution function

$$\bar{x} = x_0 x_1 \ldots x_k$$

$$f_j(\bar{x}) = L_e(x_0 \rightarrow x_1)\ T(\bar{x})\ W_e^j(x_{k-1} \rightarrow x_k)$$

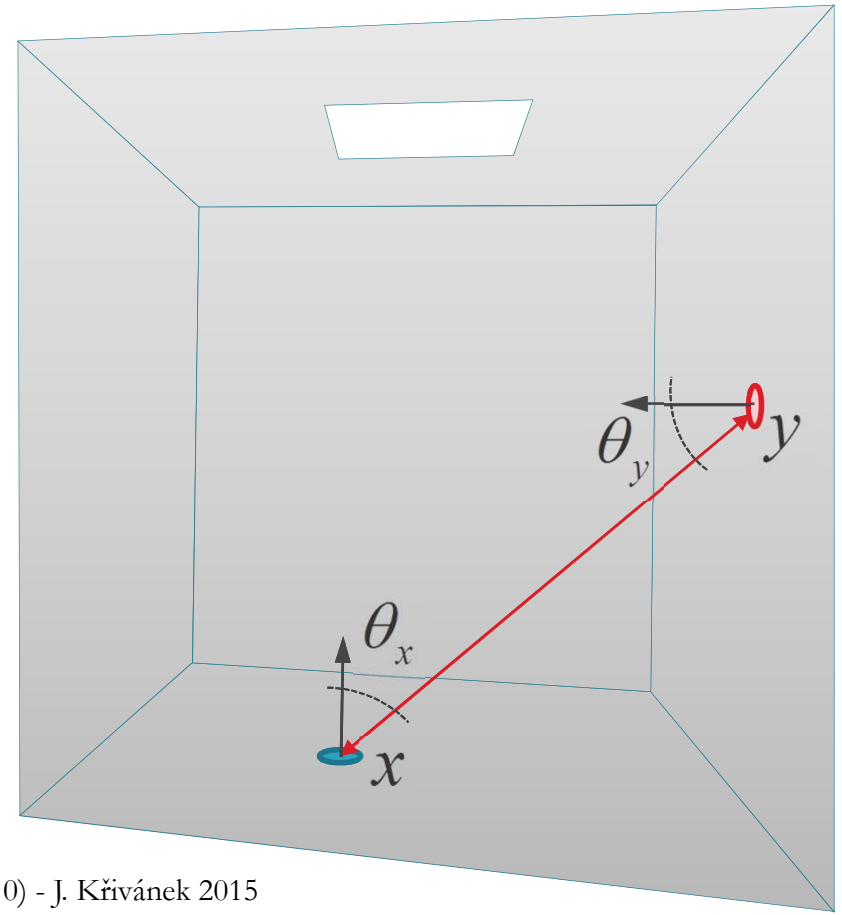
emitted radiance — path throughput — sensor sensitivity ("emitted importance")

$$T(\bar{x}) = G(-) f_r(-) \cdots f_r(-) G(-)$$



Geometry term

$$G(x \leftrightarrow y) = \frac{|\cos\theta_x||\cos\theta_y|}{\|x-y\|^2} V(x \leftrightarrow y)$$



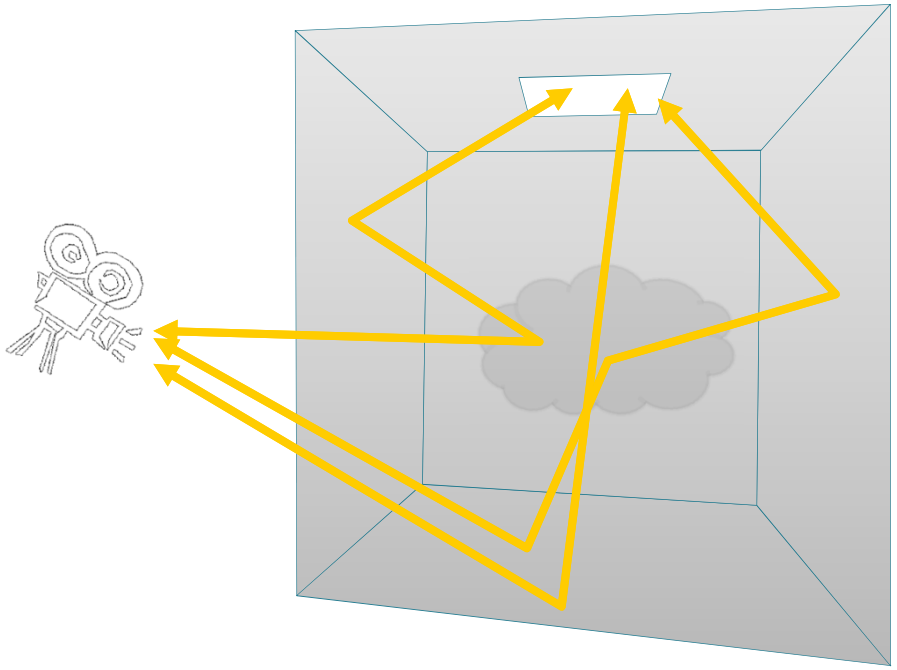
Path integral formulation

$$I_j = \int_\Omega f_j(\bar{x}) \; \mathrm{d}\mu(\bar{x})$$

$$= \sum_{k=1}^{\infty} \int_{M^{k+1}} f_j(x_0 \ldots x_k) \; \mathrm{d}A(x_0) \ldots \mathrm{d}A(x_k)$$

all path lengths

all possible vertex positions



Rendering :

Evaluating the path integral

Path integral

$$I_j = \int_\Omega f_j(\bar{x})\, \mathrm{d}\mu(\bar{x})$$

pixel value

all paths

contribution function

- **Monte Carlo integration**

MC evaluation of the path integral

Path integral

$$I_j = \int_\Omega f_j(\bar{x})\, \mathrm{d}\mu(\bar{x})$$

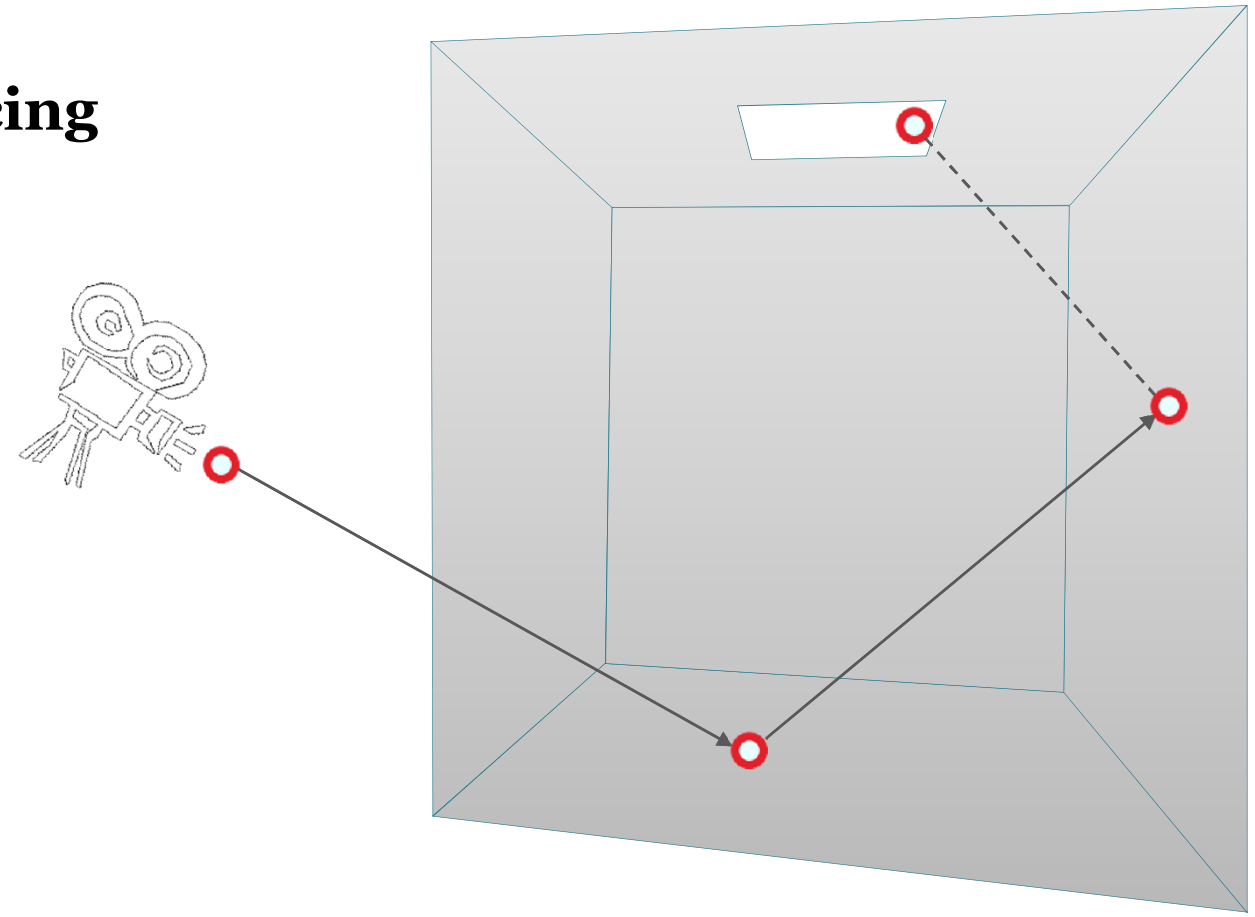
MC estimator

$$\left\langle I_j \right\rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$

- Sample path $\bar{x}$ from some distribution with PDF $p(\bar{x})$?
- Evaluate the probability density $p(\bar{x})$?
- Evaluate the integrand $f_j(\bar{x})$ ✓

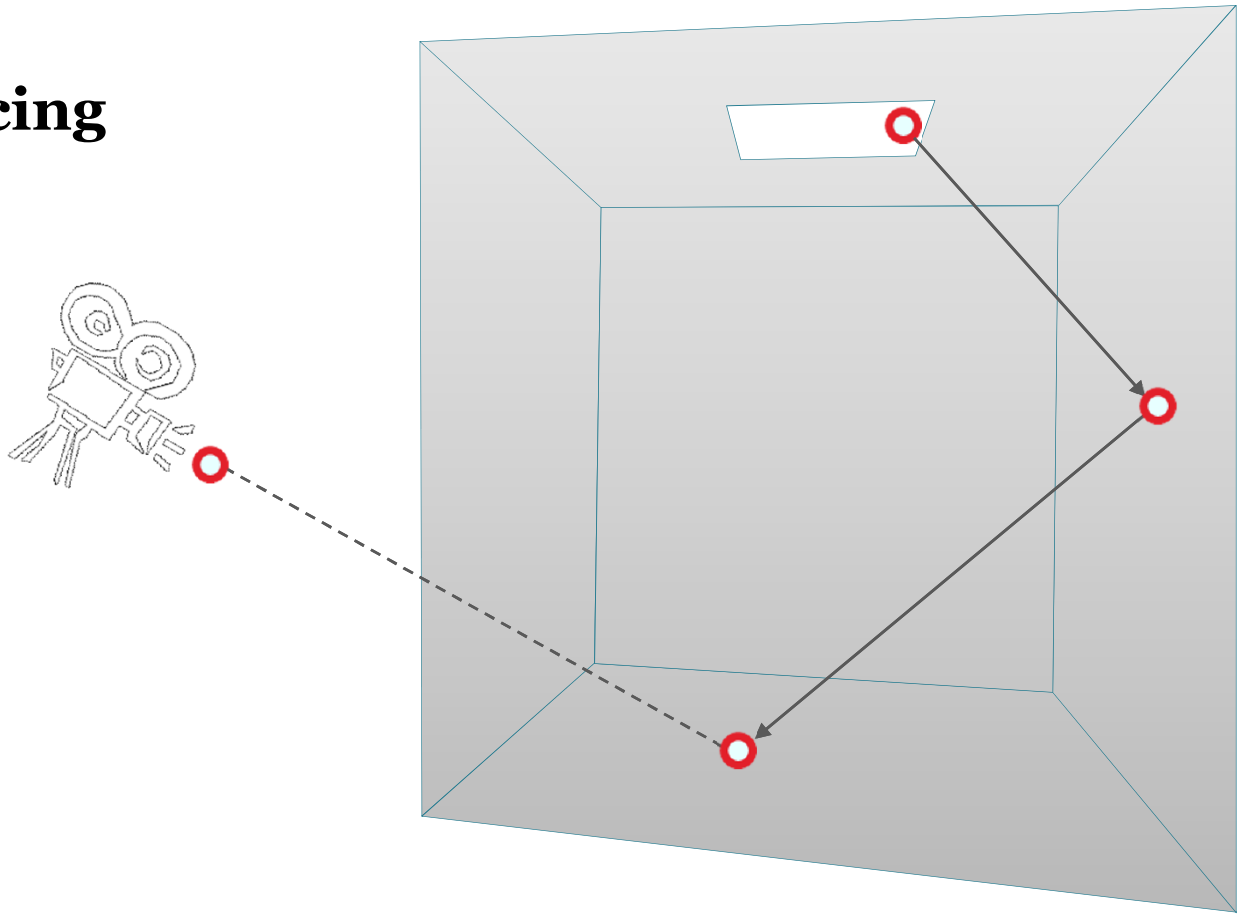
Path sampling

- Algorithms = different path sampling techniques
  - **Path tracing**



Path sampling

- Algorithms = different path sampling techniques
  - **Light tracing**



Path sampling

- Algorithms = different path sampling techniques
- **Same** general form of **estimator**

$$\langle I_j \rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$

- By taking the average of N randomly sampled paths:

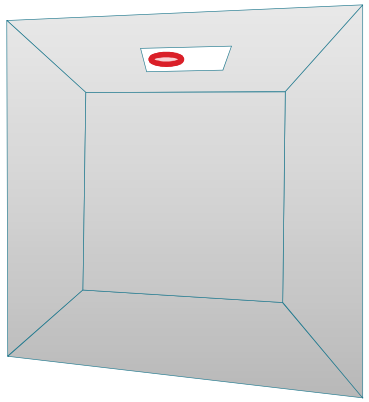
$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\bar{x}_i)}{p(\bar{x}_i)}$$

Path sampling
&
Path PDF

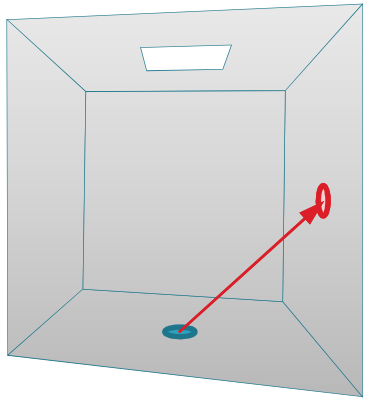
Local path sampling

- Sample one path vertex at a time

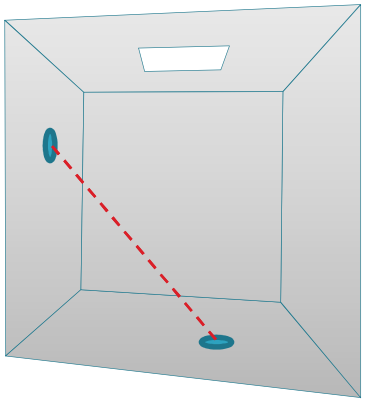
1. Sample a point on a surface, from an a priori distribution
   - lights, camera sensors, point on any surface



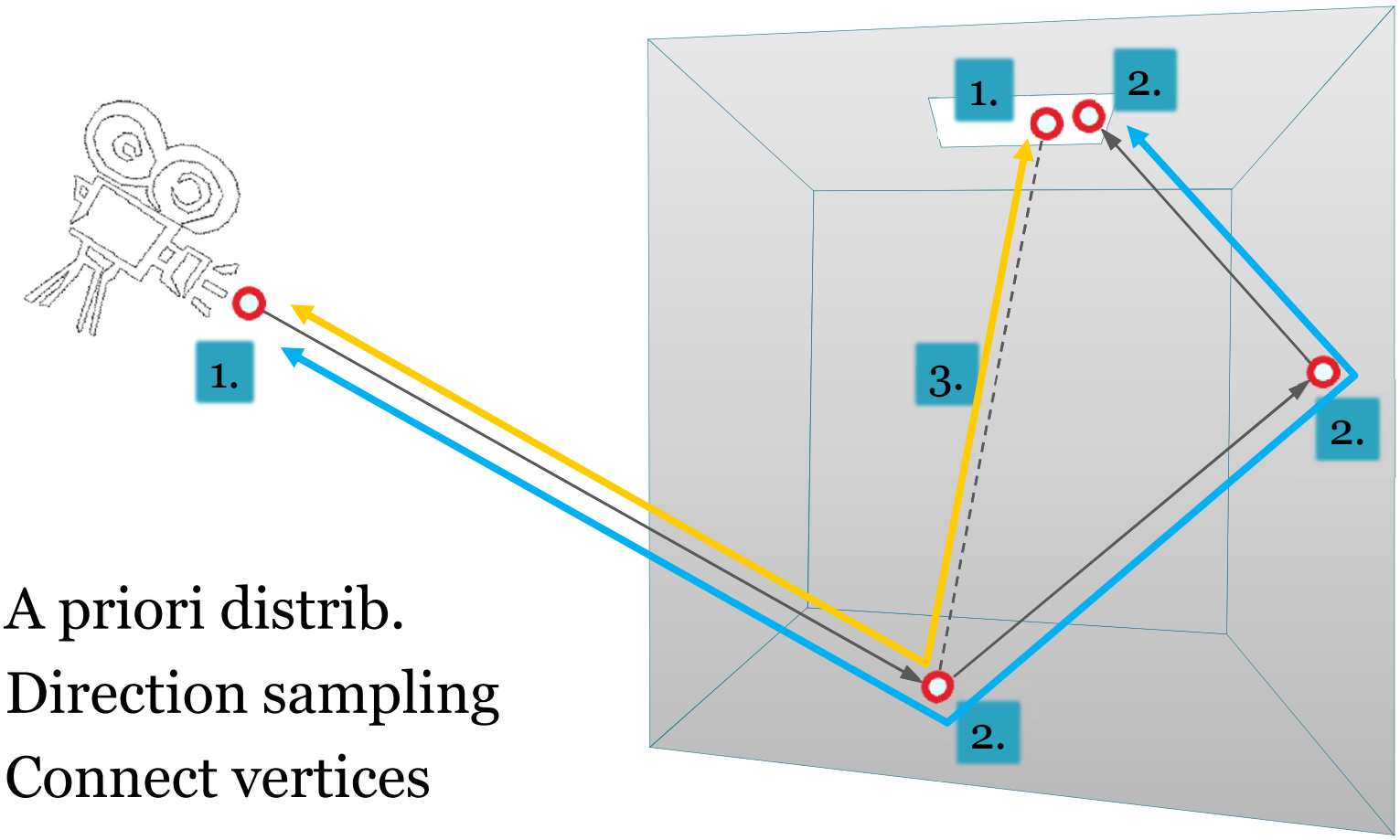
2. Sample direction from an existing vertex, according to a PDF
   - same method as used in classical path tracing



3. Connect sub-paths
   - test visibility between vertices



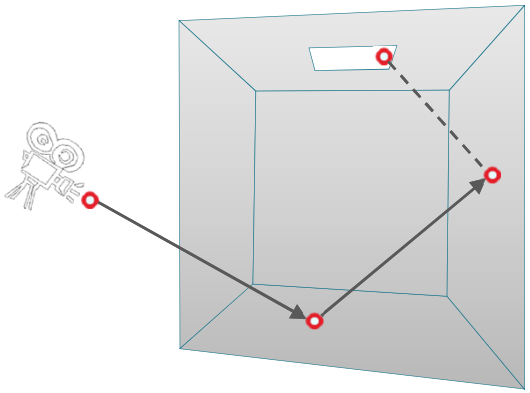
Example – Path tracing



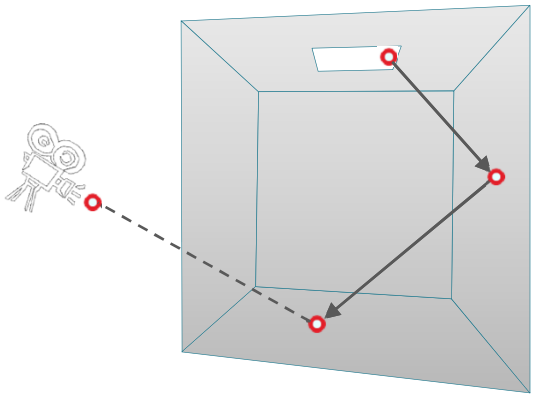
1. A priori distrib.
2. Direction sampling
3. Connect vertices

Use of local path sampling

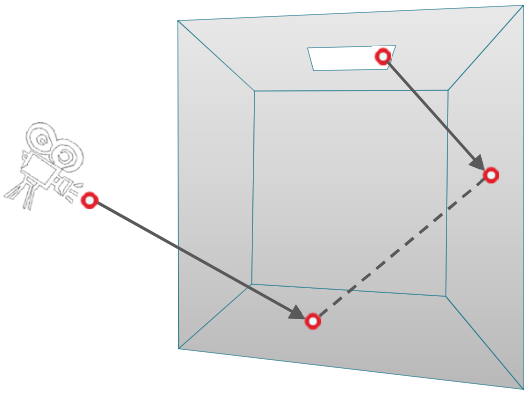
Path tracing



Light tracing



Bidirectional path tracing



Probability density function (PDF)

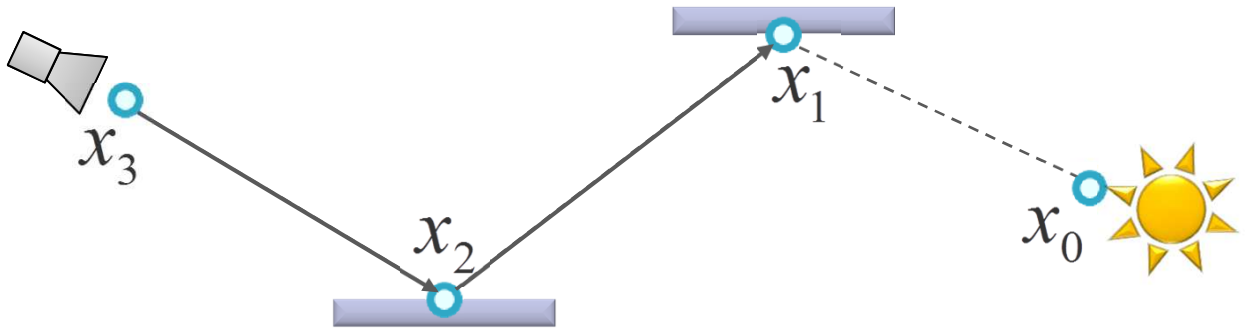
path PDF

$$p(\bar{x}) = p(x_0,\ldots,x_k) = p(x_3)\,p(x_2 \mid x_3)\,p(x_1 \mid x_2)\,p(x_0)$$

joint PDF of path vertices

product of (conditional) vertex PDFs

Path tracing example:



Summary

Path integral

$$I_j = \int_\Omega f_j(\bar{x})\, \mathrm{d}\mu(\bar{x})$$

pixel value — all paths — contribution function

MC estimator

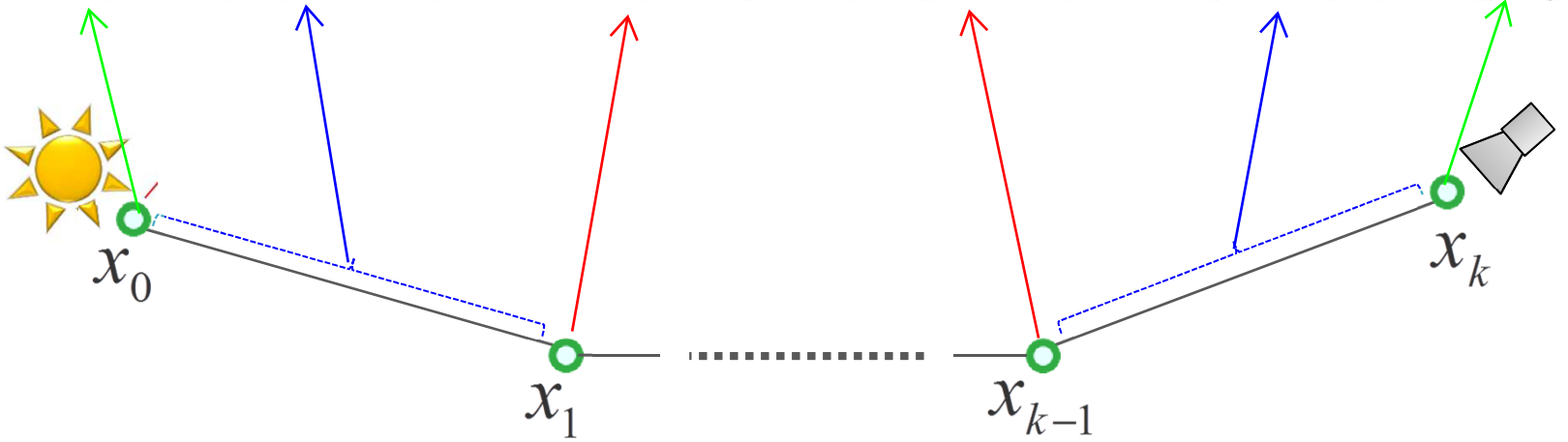
$$\left\langle I_j \right\rangle = \frac{f_j(\bar{x})}{p(\bar{x})}$$

path pdf — sampled path

$$\bar{x} = x_0 \ldots x_k$$

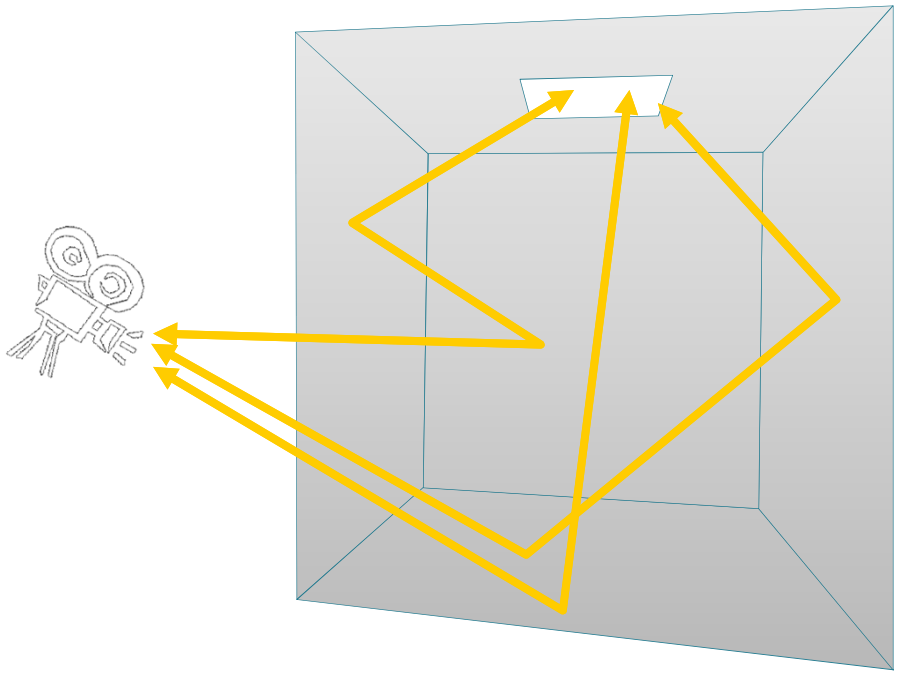
$$p(\bar{x}) = p(x_0) \ldots p(x_k)$$

$$f_j(\bar{x}) = L_e(x_0 \to x_1) G(x_0 \leftrightarrow x_1) f_r(x_0 \to x_1 \to x_2) \cdots f_r(x_{k-2} \to x_{k-1} \to x_k) G(x_{k-1} \leftrightarrow x_k) W_e^j(x_{k-1} \to x_k)$$



Summary

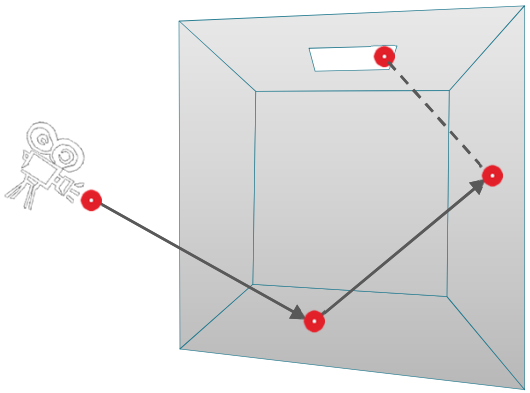
- **Algorithms**
  - different path sampling techniques
  - different path PDF



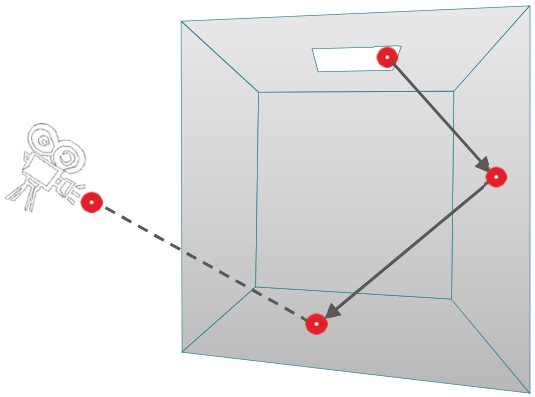
Bidirectional path tracing

Bidirectional path tracing

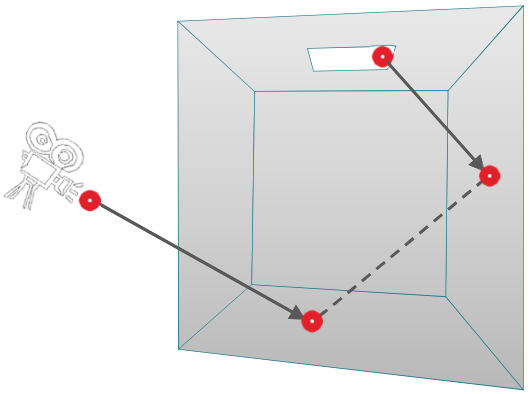
Path tracing



Light tracing

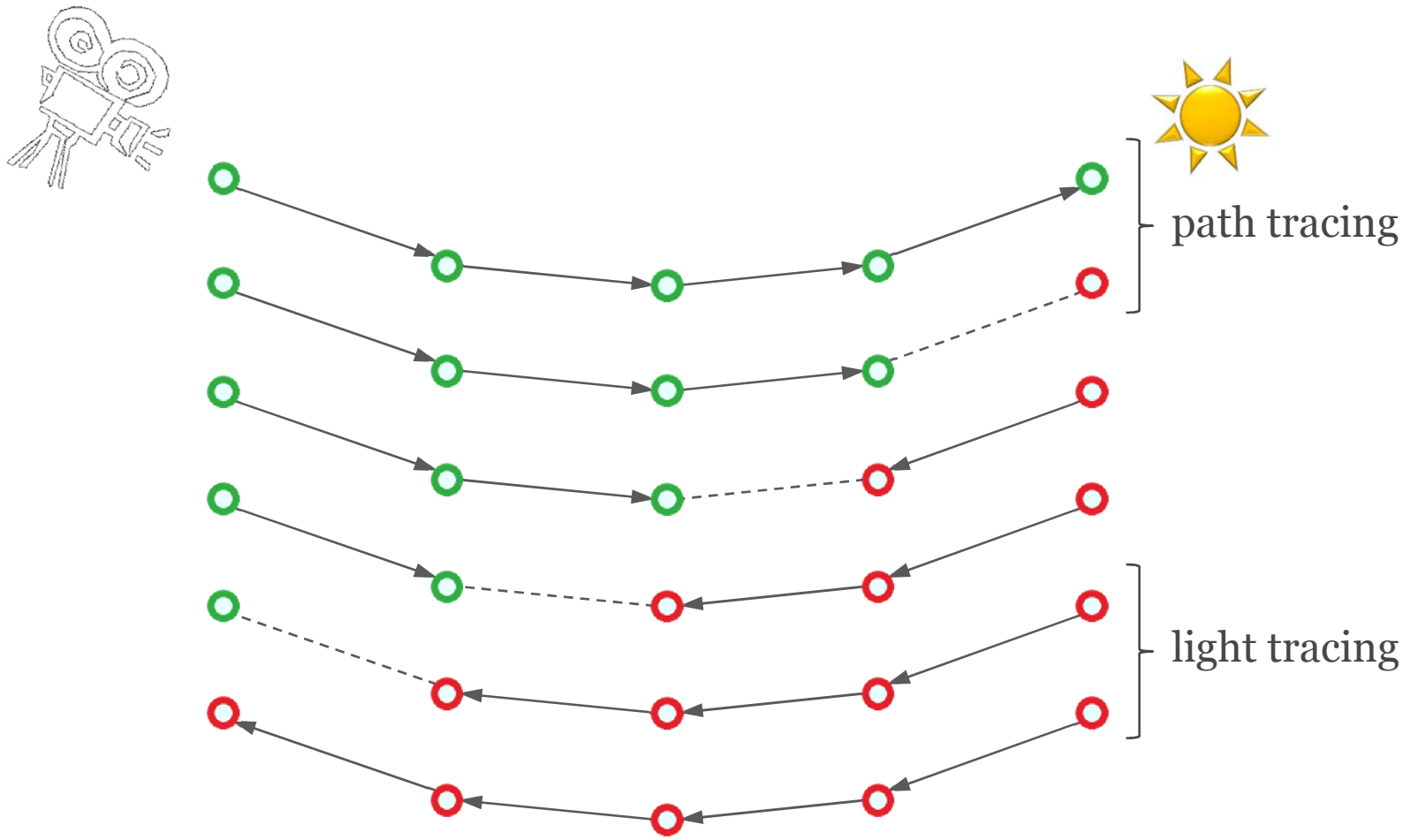


Bidirectional path sampling



All possible bidirectional techniques

vertex on a **light sub-path**
vertex on en **eye sub-path**



no single technique importance samples all the terms

Bidirectional path tracing

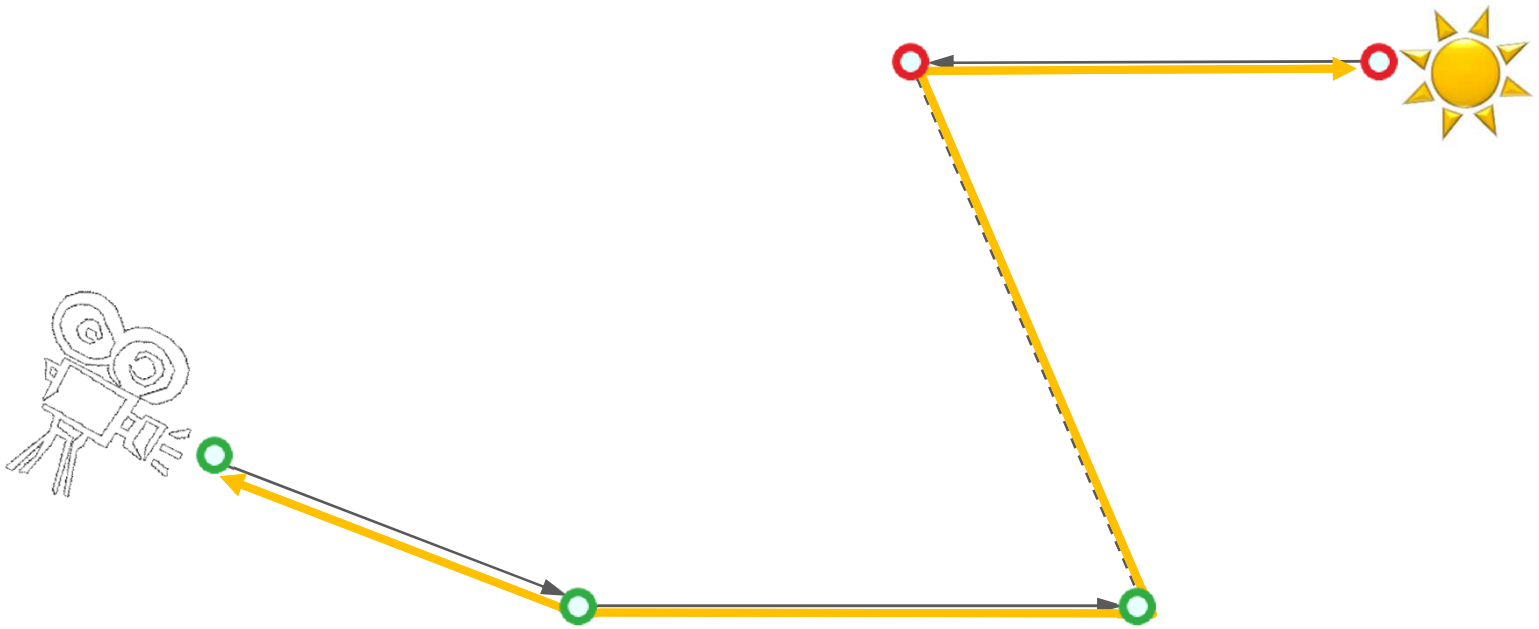
- Use **all** of the above sampling techniques
- Combine using **Multiple Importance Sampling**

- Generalizes the combined strategy for calculating direct illumination in a path tracer

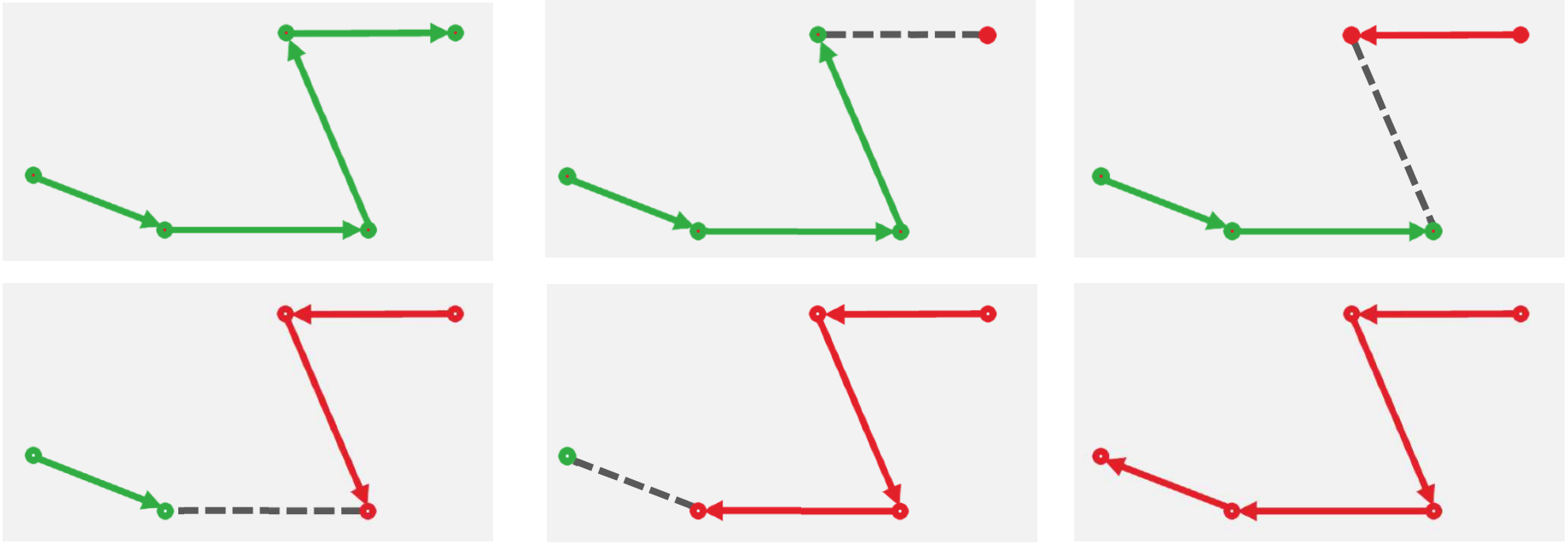
  - **PT**: Different strategies for sampling a direction toward a light source

  - **BPT:** Different strategies for sampling entire light transport paths

Naive BPT



MIS weight calculation



Sampling techniques in BPT

Example: Four techniques for $k = 2$

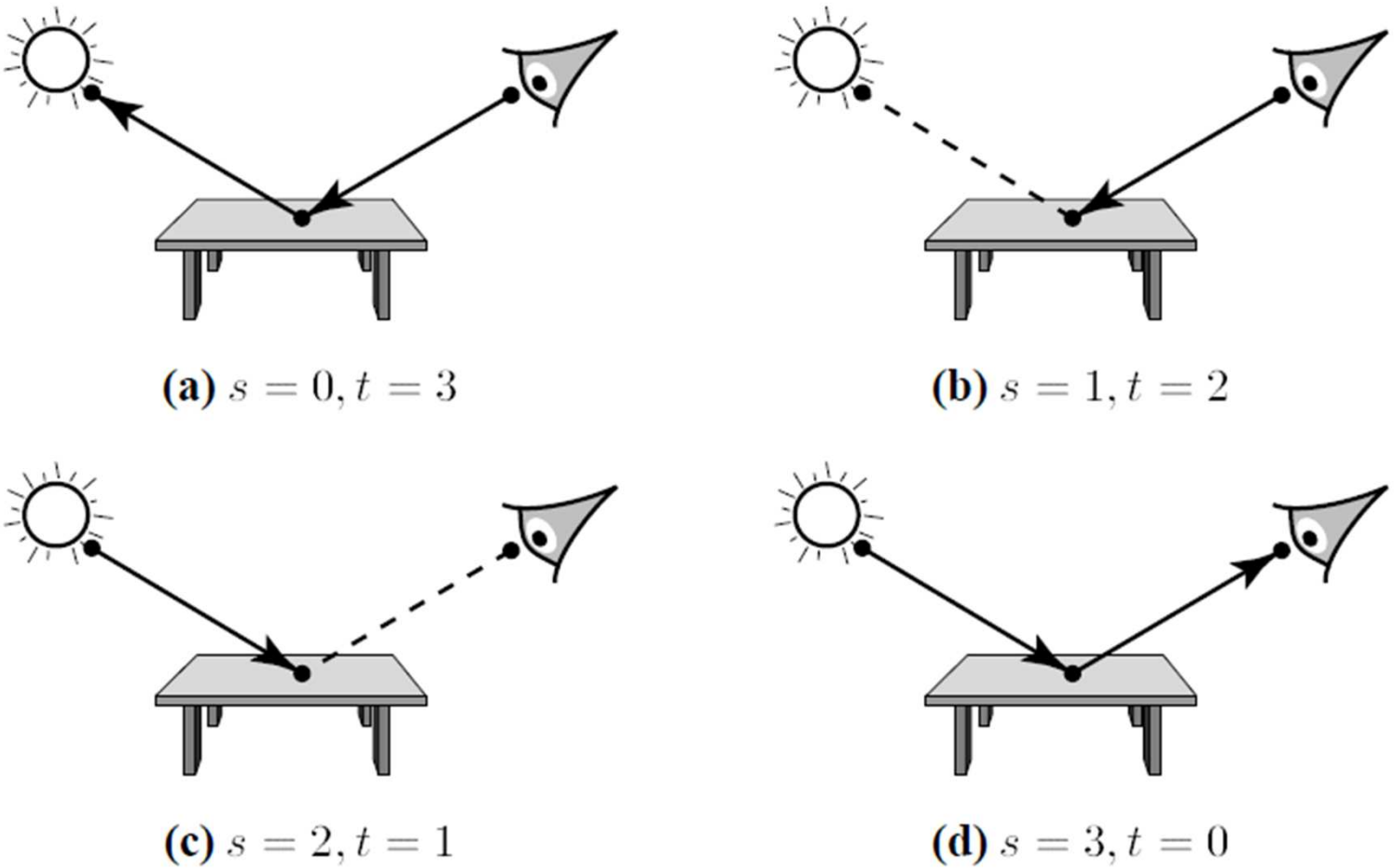


Image: Eric Veach

Sampling techniques in BPT

- Sub-path with t vertices sampled from the camera
- Sub-path with s vertices sampled from the light sources
- Connection segment of length 1
- Total path length: $k = s + t - 1$ (number of **segments**)

- In BPT, there are $k+2$ way to generate a path of length k

Sampling techniques in BPT

- Each path sampling technique has a different **probability density $p_{s,t}$**
- Each techniques is efficient at sampling different kinds of lighting effects
- All of them estimate the **same integral**

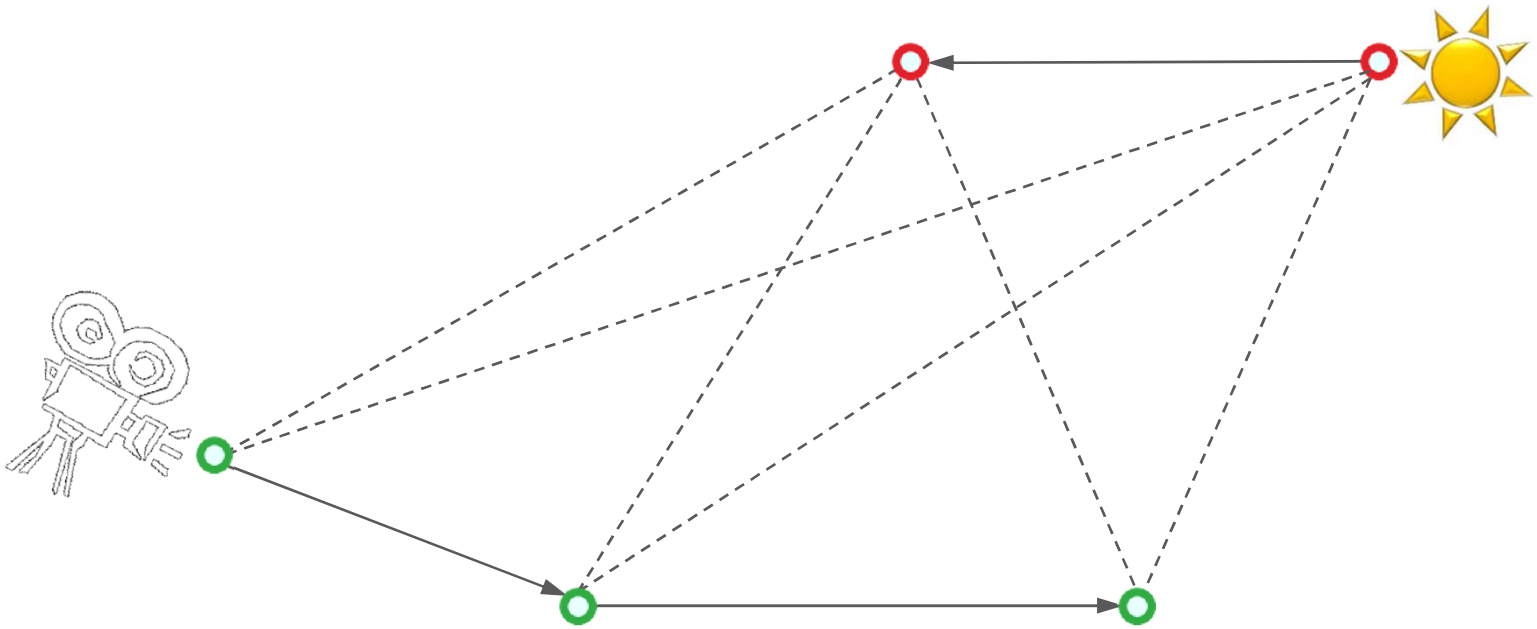
Combination of path sampling techniques

- Combined estimator (MIS)

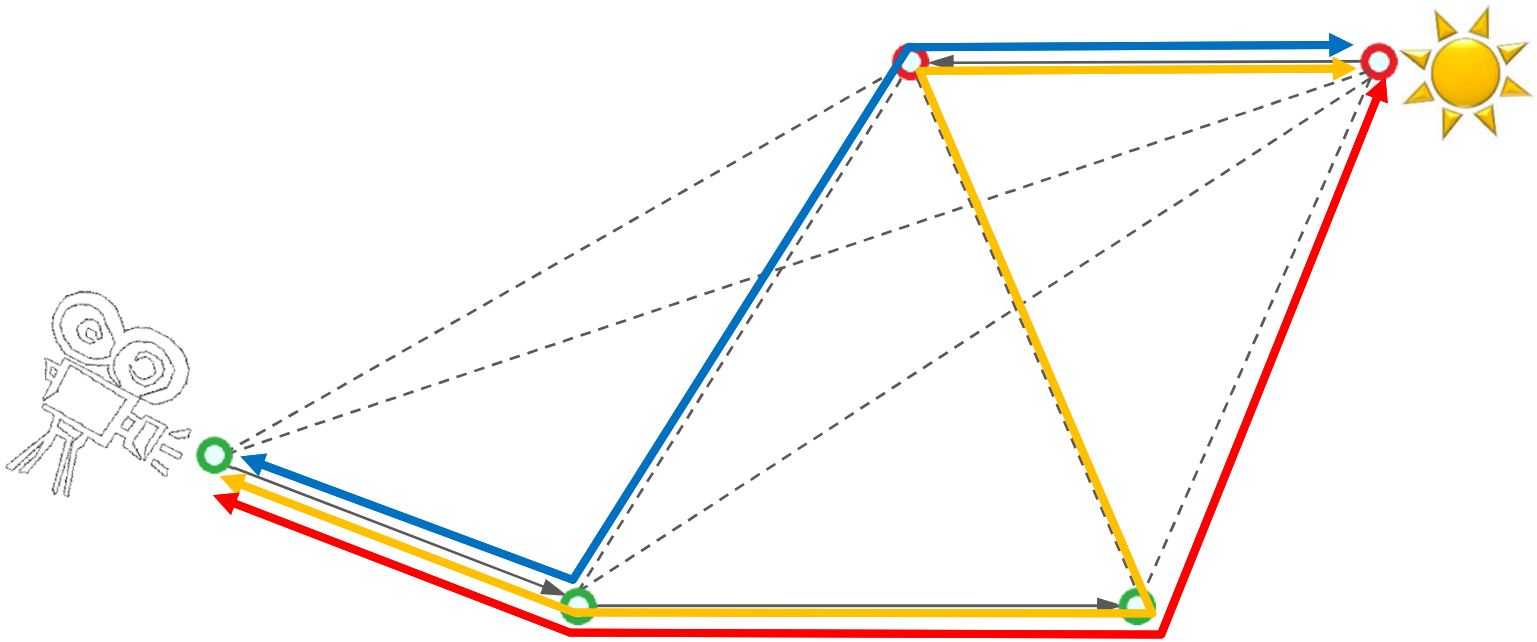
$$F = \sum_{s \geq 0} \sum_{t \geq 0} w_{s,t}(\bar{x}_{s,t}) \frac{f_j(\bar{x}_{s,t})}{p_{s,t}(\bar{x}_{s,t})}$$

MIS weights
(e.g. the balance heuristic)

BPT implementation in practice



BPT implementation in practice



BPT implementation in practice

- Sample a sub-path of a random length starting **from light sources**

$$\mathbf{y}_0 \cdots \mathbf{y}_{n_L-1}$$

- Sample a sub-path of random length starting **from the camera**

$$\mathbf{z}_{n_E-1} \cdots \mathbf{z}_0$$

- Connect each **prefix of a sub-path from light** with each **suffix of a sub-path from the camera**

$$\bar{x}_{s,t} = \mathbf{y}_0 \cdots \mathbf{y}_{s-1}\, \mathbf{z}_{t-1} \cdots \mathbf{z}_0$$

Results

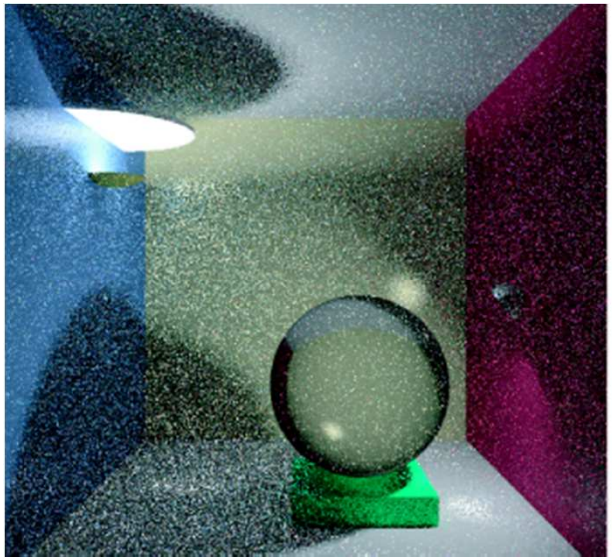
BPT, 25 samples per pixel

PT, 56 samples per pixel

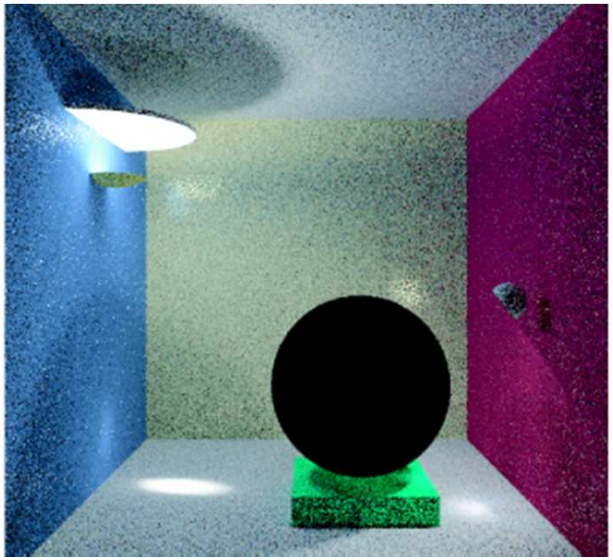
Images: Eric Veach

s / t = number of vertices on the sub-path from light / camera

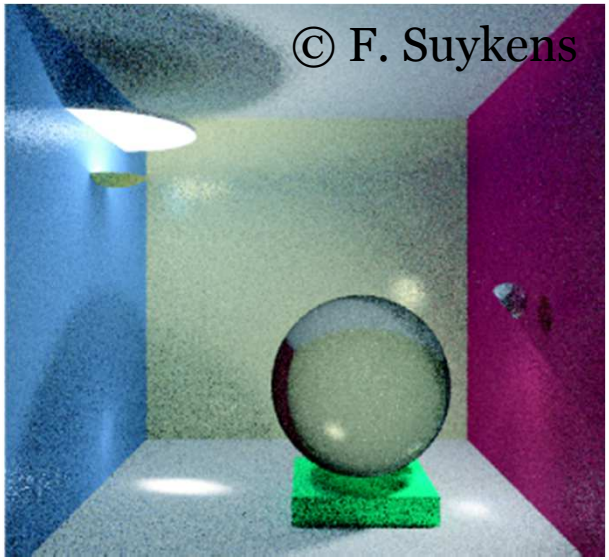
Algorithm comparison again



Path tracing

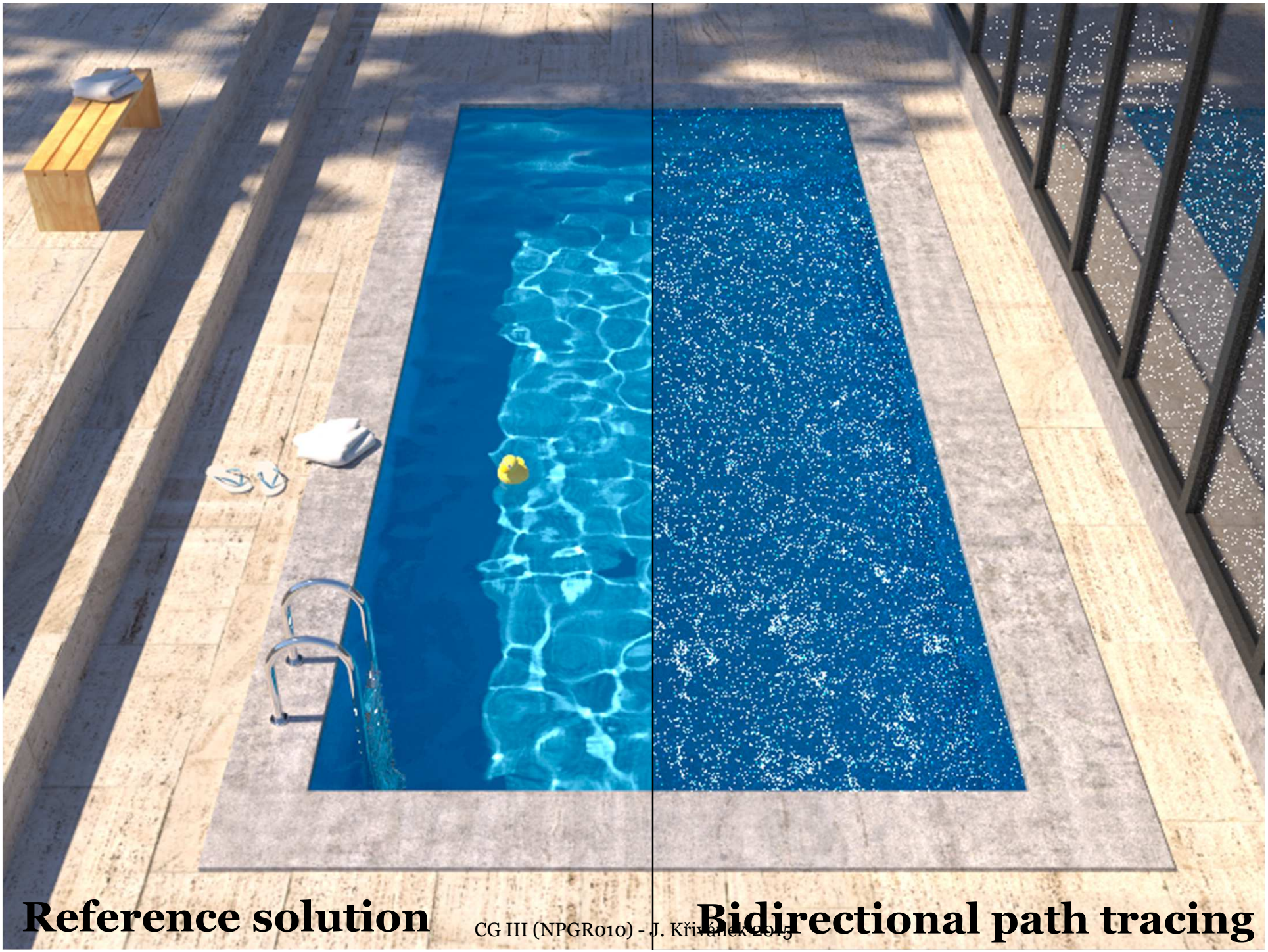


Light tracing



Bidirectional path tracing

LIMITATIONS OF LOCAL PATH SAMPLING


Reference solution
Bidirectional path tracing

Bidirectional path tracing (30 min)

Literature

E. Veach: Robust Monte Carlo methods for light transport simulation, PhD thesis, Stanford University, 1997, pp. 219-230, 297-317
http://www.graphics.stanford.edu/papers/veach_thesis/