



Recursive Ray Tracing

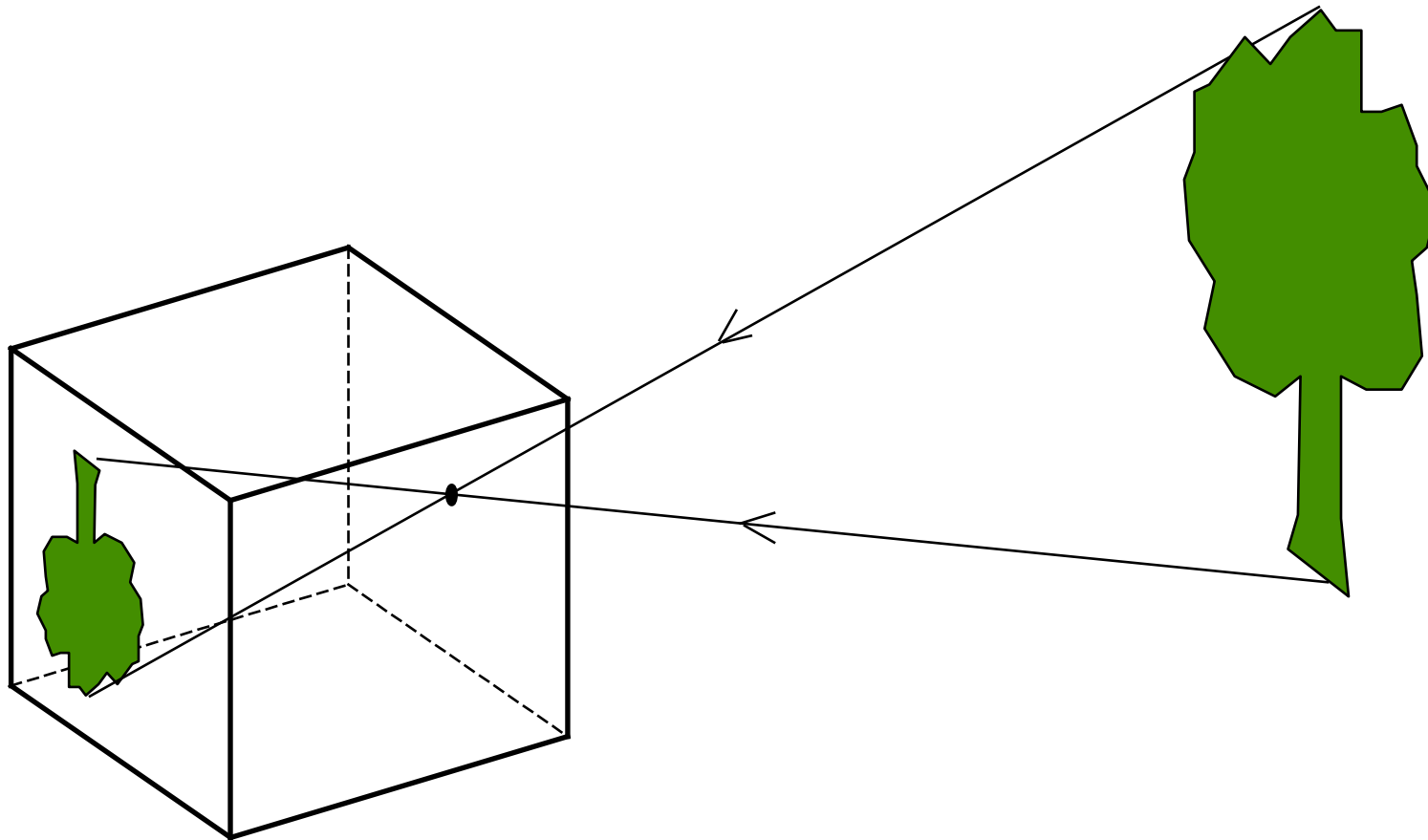
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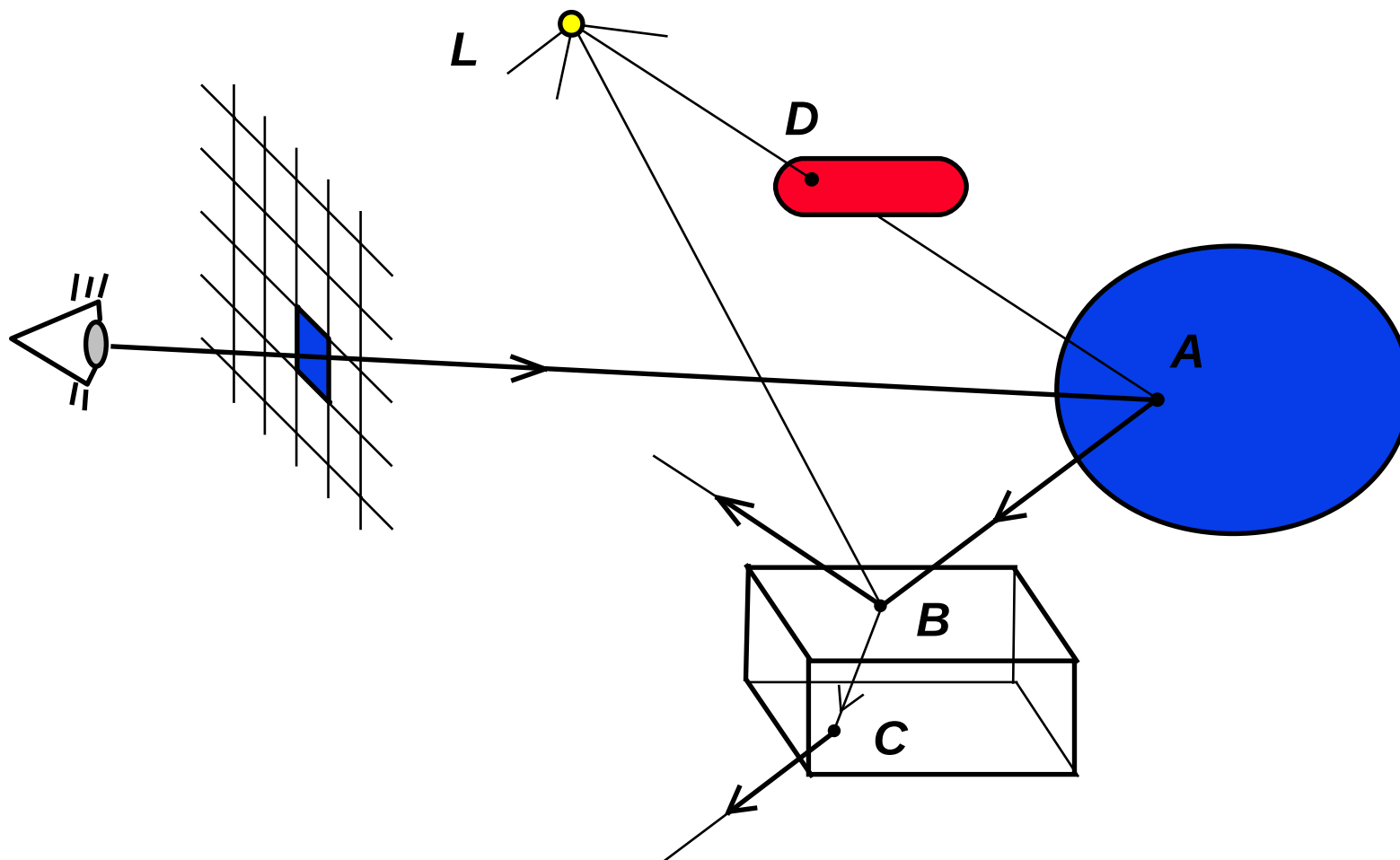
<http://cgg.mff.cuni.cz/~pepca/>



Pinhole Camera Model

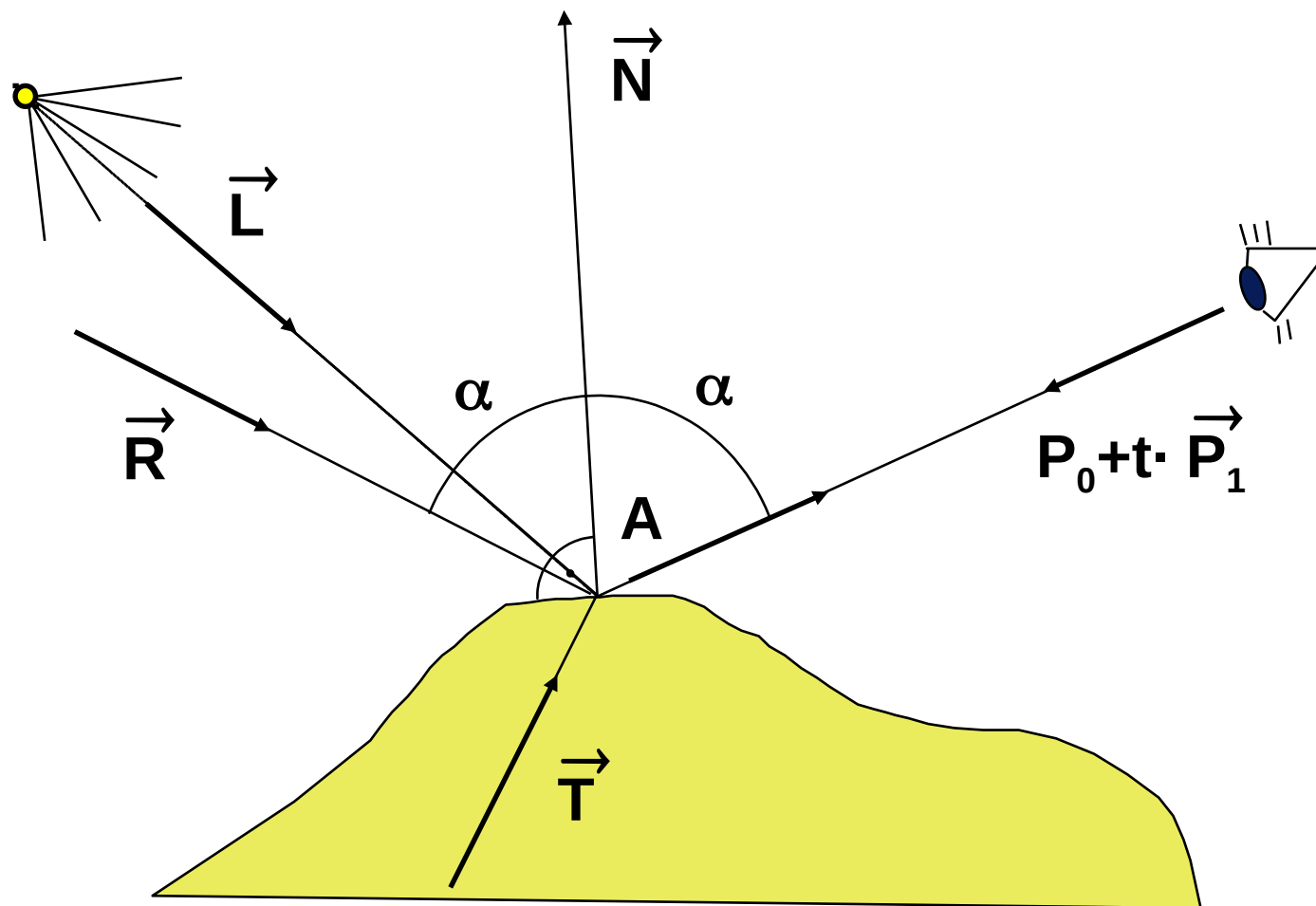


Backward / Reverse Ray Tracing





Lighting Computations





Recursive Implementation

```
function Trace ( P0, P1 : Point3D; depth : integer ) : RGB;
    { P0..ray begining, P1..direction, depth..number of reflections }
var A, R, T : Point3D;           { temporary points and vector }
    B : RGB;                      { resulting color }
begin
    A := Intersection(Scene,P0,P1) { ray intersection with scene }
    if A==0 then Result:= Background { no intersection }
    else
        begin { ray intersects an object }
            B := 0;
            for i := 1 to N do { contributions from N light sources }
                if Intersection(Scene,A,L[i]-A) == 0 then B := B + Light(A,L[i]);
            depth := depth + 1;
            if depth <= maxdepth then { end of recursion }
                begin
                    if "A has nonzero reflection coef kR" then
                        begin
                            "calculate reflected intensity" { reflected ray R}
                            B := B + kR * Trace(A,R,depth);
                        end;
                end;
```



Recursive Implementation

```
if "A has nonzero transm. coef kT" then  
  begin  
    "calculate refracted intensity" { refracted ray T }  
    B := B + kT * Trace(A,T,depth);  
  end;  
end;  
Result := B;           { return value }  
end;  
end;
```



Recursion Management

- ① **static** – limited by a constant (not suitable for scenes with many mirrors)
- ② **dynamic** – according to performance of the ray
 - ➡ „**performance**” is the percentage the ray can still contribute to the colour of a given pixel (primary rays: 100%)
 - ➡ Limit on the „performance” constant (e.g. 2-10%)
- ③ **Combined** – limit on the recursion depth, and the „performance” of the ray

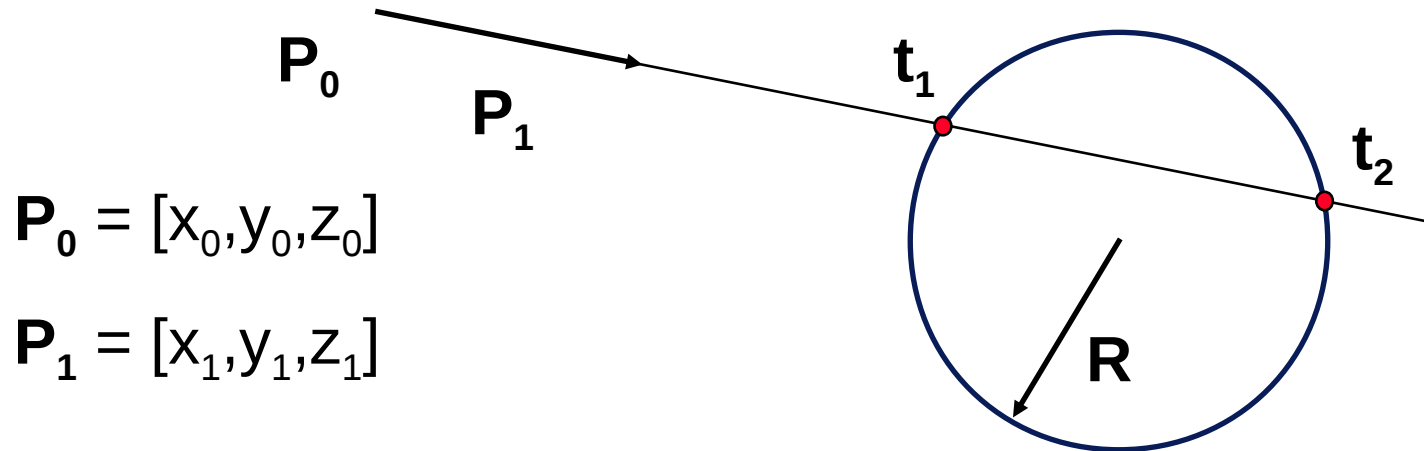


Intersection Computations

- ① Intersection coordinates (or „infinity”)
- ② ID of the solid (surface) that was hit
- ③ Normal vector at the intersection
- ➔ **Time-consuming operation** (80-90% of render time)
 - Acceleration techniques extremely important
- ◆ **Analytical solution** (sphere, cylinder, cube, ..)
- ◆ **Numerical solution** (subdivision surfaces, higher order surfaces, rotational surfaces, ..)



Ray-Sphere Intersections



$$\mathbf{P}_0 = [x_0, y_0, z_0]$$

$$\mathbf{P}_1 = [x_1, y_1, z_1]$$

➔ Ray: $\mathbf{P}_0 + t \mathbf{P}_1, \quad t > 0$ (1)

➔ Sphere (at origin): $\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2 - \mathbf{R}^2 = 0$ (2)

➔ When substituting (1) into (2) we obtain a **quadratic equation** for t :

$$t^2 (\mathbf{x}_1^2 + \mathbf{y}_1^2 + \mathbf{z}_1^2) + 2t (\mathbf{x}_0 \mathbf{x}_1 + \mathbf{y}_0 \mathbf{y}_1 + \mathbf{z}_0 \mathbf{z}_1) + \mathbf{x}_1^2 + \mathbf{y}_1^2 + \mathbf{z}_1^2 - \mathbf{R}^2 = 0$$

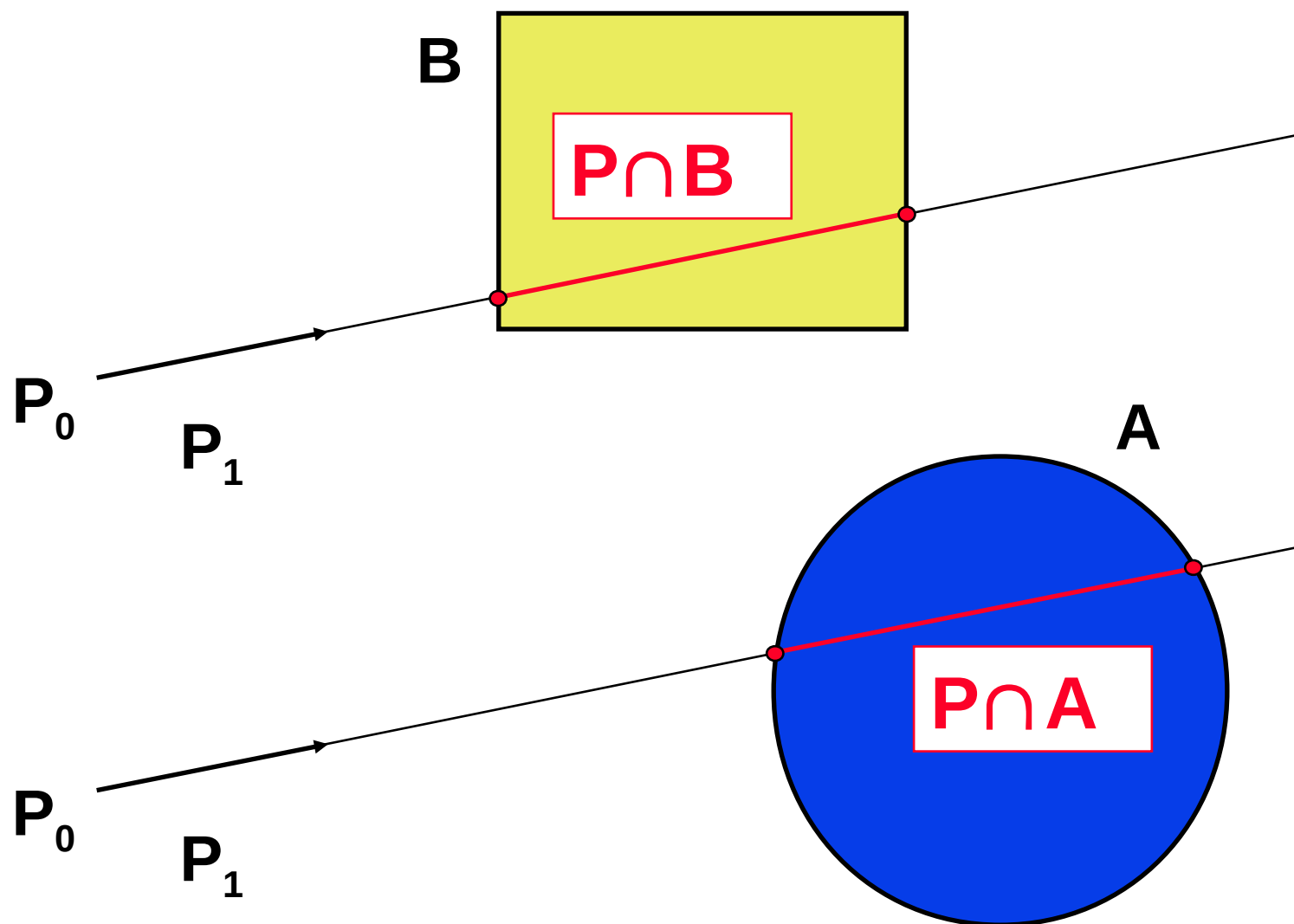


Ray Intersections with CSG

- ◆ For **elementary solids**, intersections can be calculated
 - Start and end of ray traversal through a solid body
- ◆ **Set theoretic operations** on all intersections along the ray:
 - Distributive: $\mathbf{P} \cap (\mathbf{A} - \mathbf{B}) = (\mathbf{P} \cap \mathbf{A}) - (\mathbf{P} \cap \mathbf{B})$
 - The usual ray-object intersection is an interval
- ◆ **Geometric transformations**:
 - The inverse transformation is applied to the ray

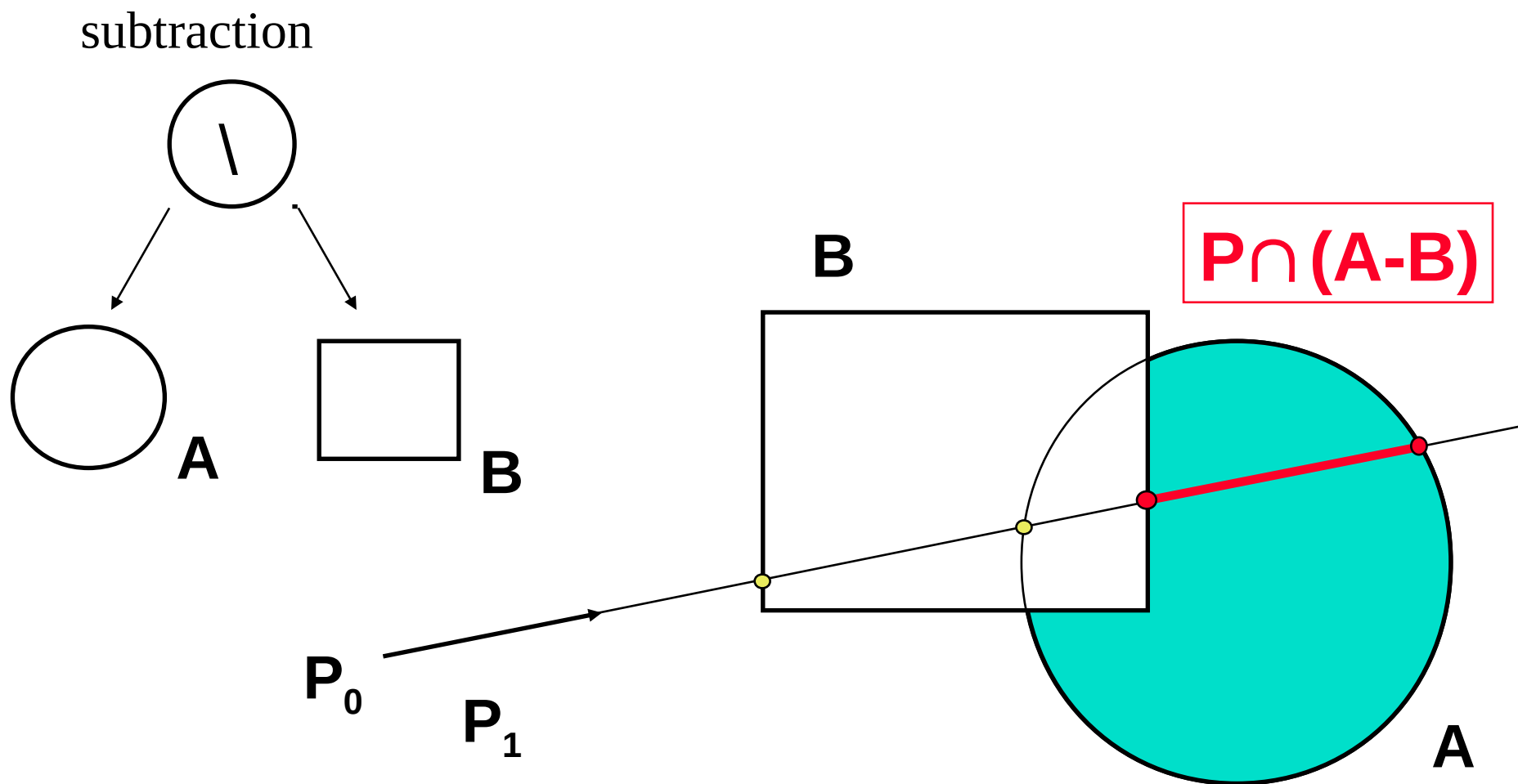


Intersection $P \cap A$, $P \cap B$



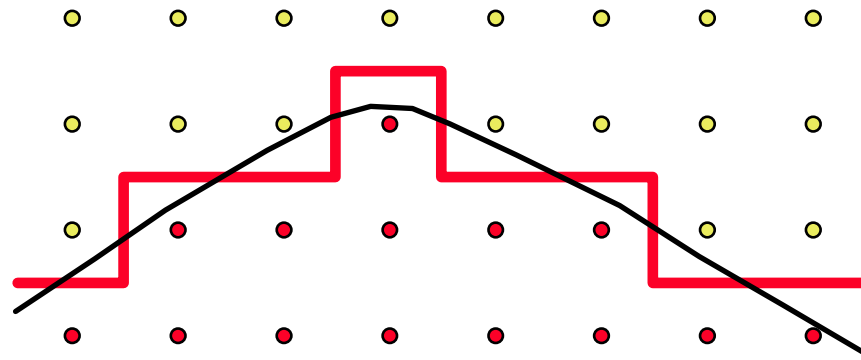


Intersection: $P \cap (A-B)$





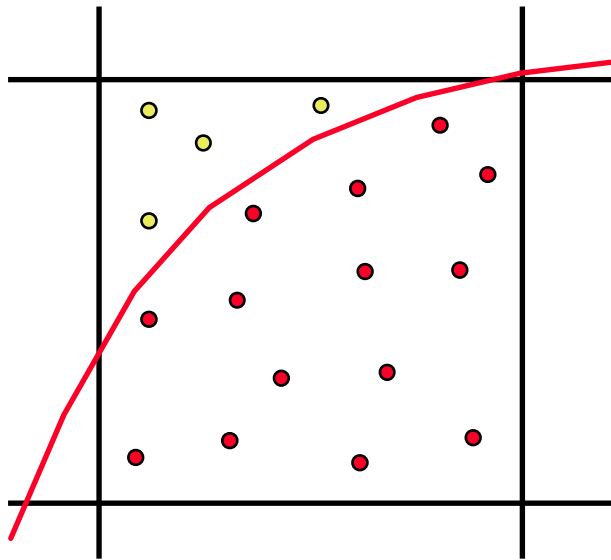
Anti-aliasing



- ◆ Only one ray per pixel leads to „aliasing”
 - Jagged edges
 - Interference
- ◆ **Increased resolution** only solves the problem partially (and at great expense)



Multiple Sampling



- ◆ Multiple rays are shot into each pixel
- ◆ The resulting colour is obtained via arithmetic mean
- ◆ Transitions are softer (no jaggies)
- ◆ The rays should cover the pixel area evenly, but not regularly!

End



Further information:

- **A. Glassner: *An Introduction to Ray Tracing*, Academic Press, London 1989, 1-31**
- **Jiří Žára a kol.: *Počítačová grafika*, principy a algoritmy, 374-378**