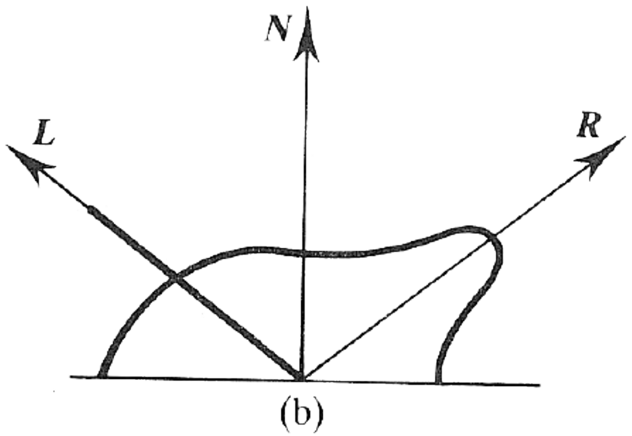
Computer Graphics

- Shading -

Empirical BRDF Approximation

- Purely heuristic model
  - Initially without units (values $\in [0,1]$)
- $L_r = L_{r,a} + L_{r,d} + L_{r,s}$ $(+ L_{r,m} + L_{r,t})$
- $L_{r,a}$: Ambient term
  - Approximate indirect illumination
- $L_{r,d}$: Diffuse term (Lambert)
  - Uniform reflection
- $L_{r,s}$: Specular term
  - Mirror-reflection on a rough surface
- $L_{r,m}$: Perfect reflection
  - *Only possible with Ray-Tracing*
- $L_{r,t}$: Perfect transmission
  - *Only possible with Ray-Tracing*



Phong Illumination Model

- **Extended light sources: *l* point light sources**

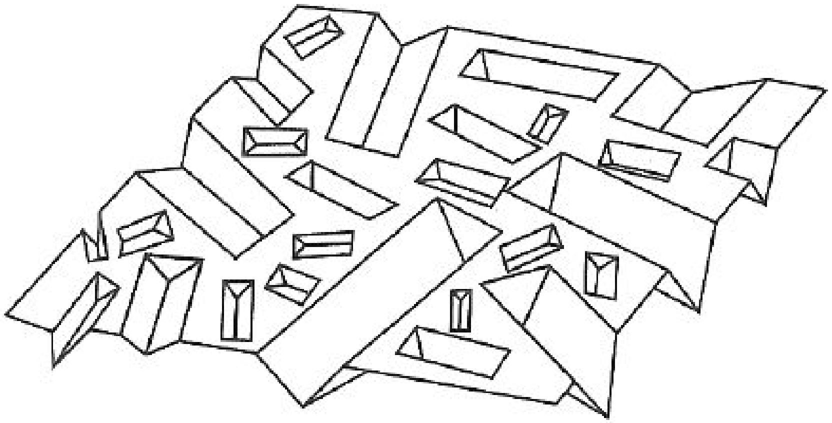
$$\mathrm{L_r} = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \qquad \text{(Phong)}$$

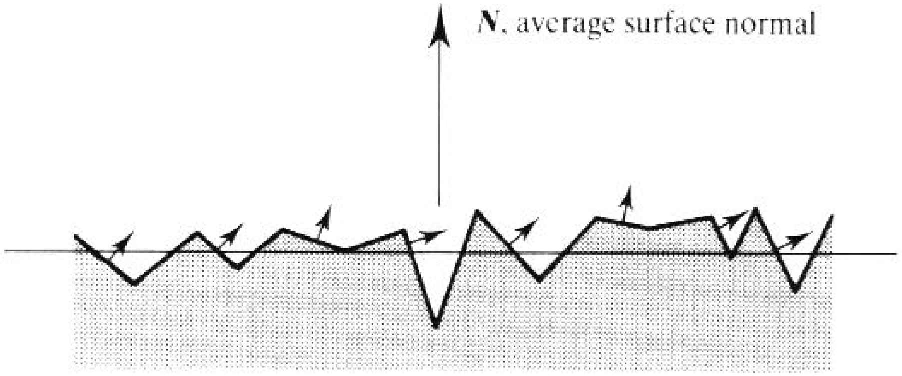
$$\mathrm{L_r} = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \qquad \text{(Blinn)}$$

- **Color of specular reflection equal to light source**
- **Heuristic model**
  - Contradicts physics
  - Purely local illumination
    - Only direct light from the light sources
    - No further reflection on other surfaces
    - Constant ambient term
- **Often: light sources & viewer assumed to be far away**

Microfacet Model

- **Isotropic microfacet collection**
- **Microfacets assumed as perfectly smooth reflectors**
- **BRDF**
  - Distribution of microfacets
    - Often probabilistic distribution of orientation or V-groove assumption
  - Planar reflection properties
  - Self-masking, shadowing





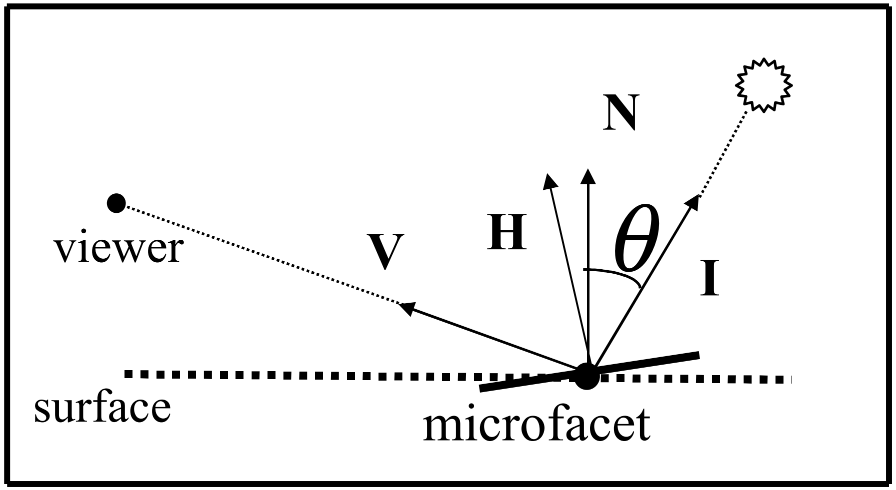
Ward Reflection Model

- **BRDF**

$$f_r = \frac{\rho_d}{\pi} + \rho_s \frac{1}{\sqrt{(I \bullet N)(V \bullet N)}} \bullet \frac{\exp(-\tan^2 \angle(H,N)/\sigma^2)}{4\pi\sigma^2}$$

σ standard deviation (RMS) of surface slope

- Simple expansion to anisotropic model (σ_x, σ_y)
- Empirical, not physics-based
- Inspired by notion of reflecting microfacets
- Convincing results
- Good match to measured data



Physics-inspired BRDFs

- **Notion of reflecting microfacet**
- **Specular reflectivity of the form**

$$f_r = \frac{D \cdot G \cdot F_\lambda(\lambda, \theta_i)}{\pi \, \underline{N} \cdot \underline{V}}$$

  - D : statistical microfacet distribution
  - G : geometric attenuation, self-shadowing
  - F : wavelength, angle dependency of reflection along mirror direction
  - $\underline{N} \cdot \underline{V}$: flaring effect at low angle of incidence
- **Cook-Torrance model**
  - F : wavelength- and angle-dependent reflection
  - Metal surfaces

Cook-Torrance Reflection Model

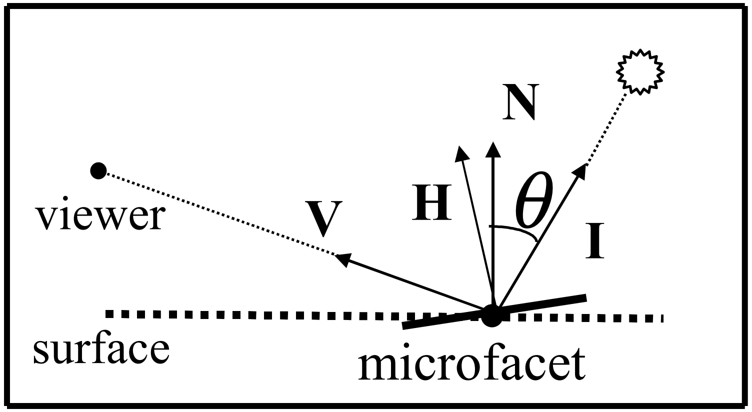
- Cook-Torrance reflectance model is based on the *microfacet* model. The BRDF is defined as the sum of a diffuse and specular components:

$$f_r = k_d \rho_d + k_s \rho_s; \qquad k_d + k_s \leq 1$$

  where k_s and k_d are the specular and diffuse coefficients.
- Derivation of the specular component ρ_s is based on a **physically derived** theoretical reflectance model

Cook-Torrance Specular Term

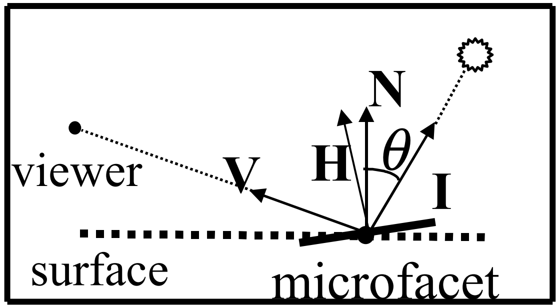
$$\rho_s = \frac{F_\lambda D G}{\pi(\underline{N} \cdot \underline{V})(\underline{N} \cdot \underline{I})}$$



- **D : Distribution function of microfacet orientations**
- **G : Geometrical attenuation factor**
  - represents self-masking and shadowing effects of microfacets
- **F_λ : Fresnel term**
  - computed by Fresnel equation
  - relates incident light to reflected light for each planar microfacet
- **N·V : Proportional to visible surface area**
- **N·I : Proportional to illuminated surface area**

Microfacet Distribution Functions

- **Isotropic Distributions** $D(\underline{\omega}) \Rightarrow D(\alpha) \quad \alpha = \mathbf{N} \cdot \mathbf{H}$



  α : angle to average normal of surface
  - Characterized by half-angle β

$$D(\beta) = \frac{1}{2}$$

- **Blinn** $D(\alpha) = \cos^{\frac{\ln 2}{\ln \cos \beta}} \alpha$

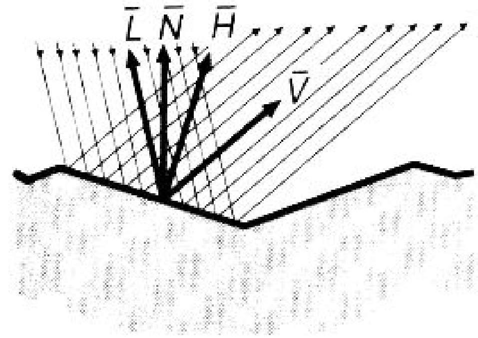
- **Torrance-Sparrow** $D(\alpha) = e^{-\left(\frac{\sqrt{2}}{\beta}\alpha\right)^2}$

- **Beckmann** $D(\alpha) = \frac{1}{4m^2 \cos^4 \alpha} e^{-[\tan \alpha / m]^2}$
  - m : root mean square
  - Used by Cook-Torrance

Geometric Attenuation Factor

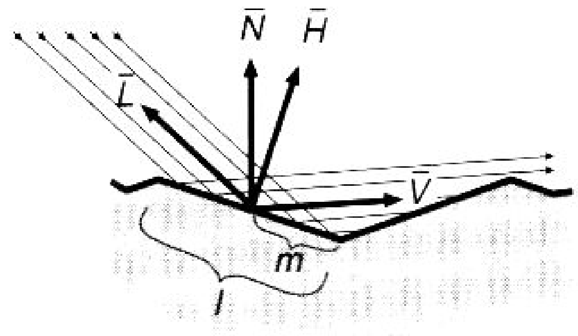
- **V-shaped grooves**
- Fully illuminated and visible

$$G = 1$$



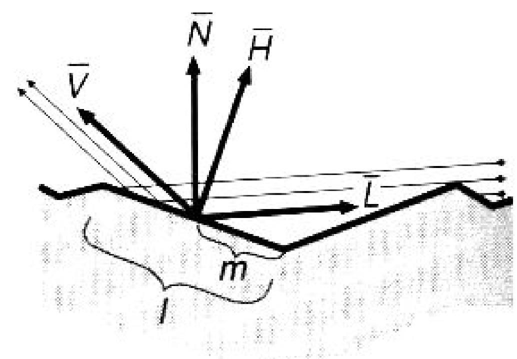
- Partial masking of reflected light

$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}$$



- Partial shadowing of incident light

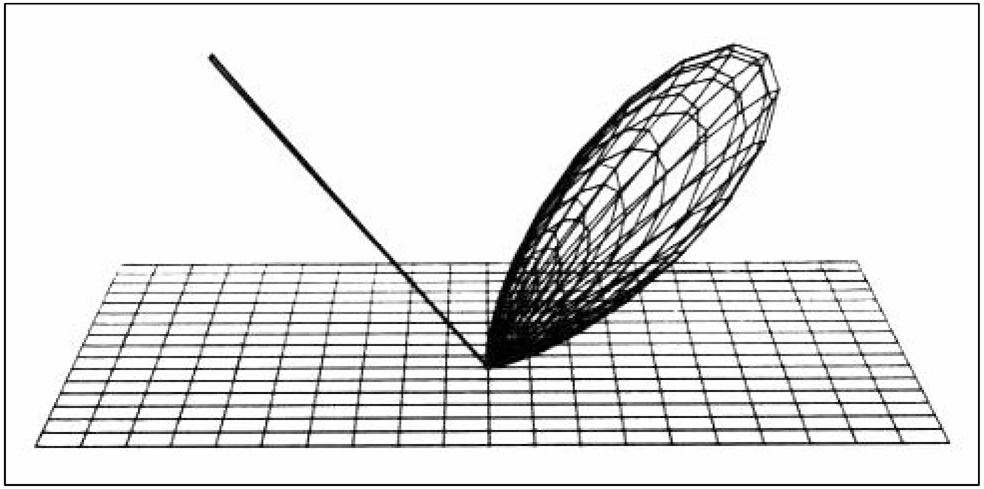
$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})}$$



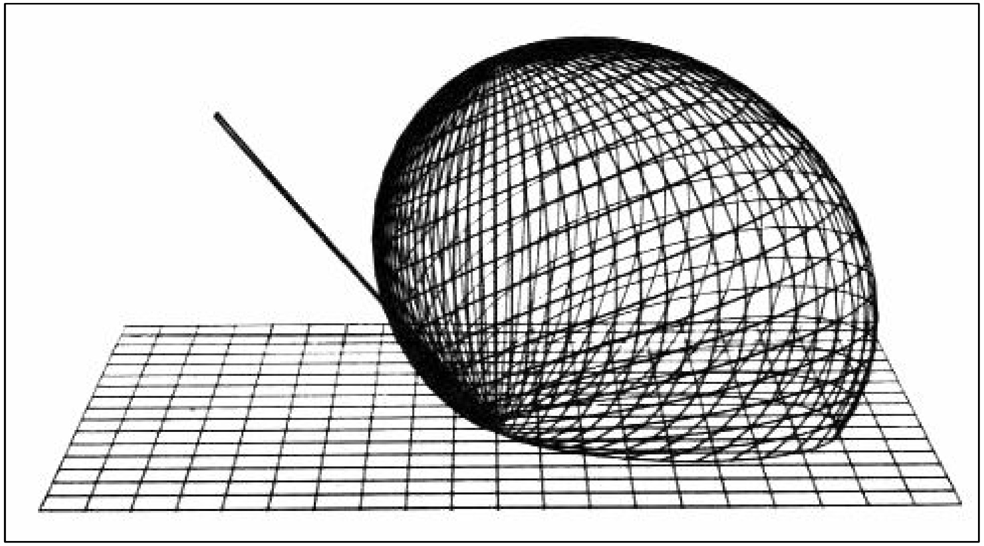
$$G = \min\left\{1, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})}\right\}$$

Beckman Microfacet Distribution Function

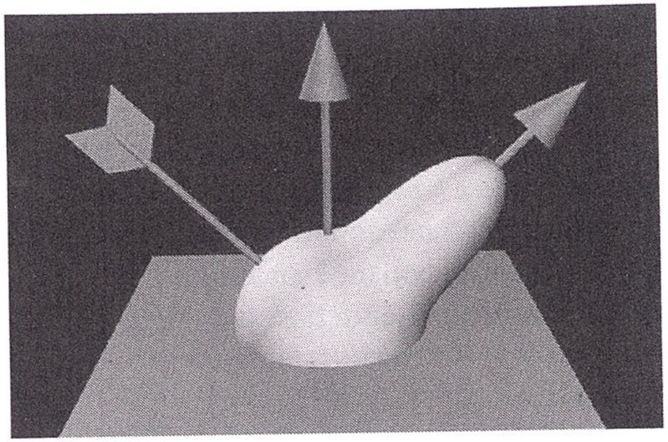
m=0.2

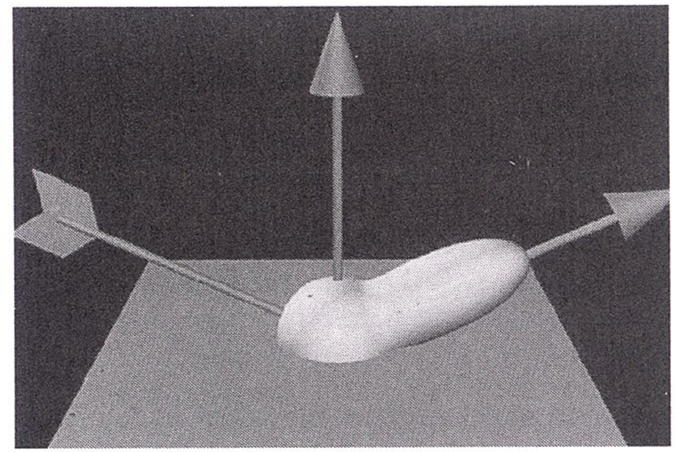


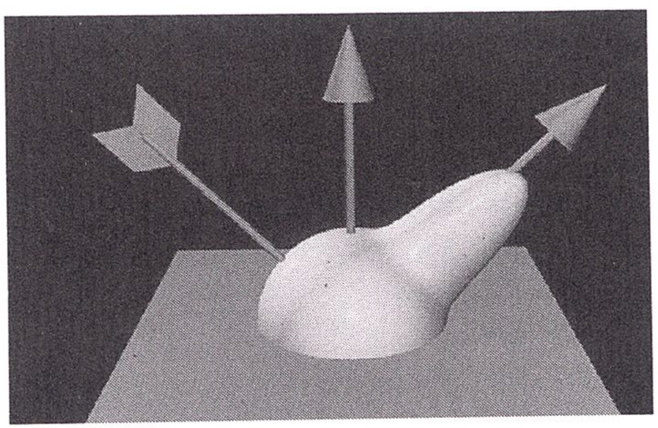
m=0.6

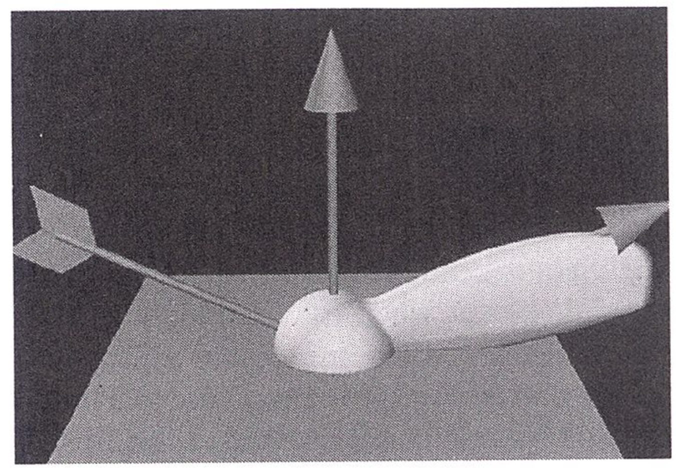


Comparison Phong vs. Torrance


(a)


(b)


(c)

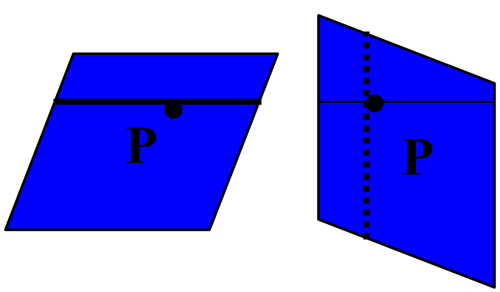

(d)

Polygon-Shading Methods

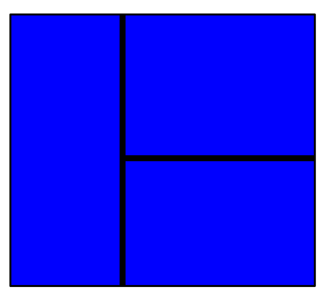
- Application of an illumination model to compute intensity for every pixel has been time consuming.
- Intensity of adjacent pixels is usually very similar (the so called shading coherence), which allows for less frequent shading evaluations.
- Each polygon can be rendered with a single intensity or intensity can be obtained at each point of the surface using an interpolation scheme:
  - ***Flat shading***, single intensity is calculated for each polygon
  - ***Gouraud shading*** (per vertex shading), intensity calculated at vertices is interpolated across the surface
  - ***Phong shading*** (per pixel shading), normal vectors are calculated at vertices; then normal vectors are interpolated across the surface and an illumination model using these normal vectors is applied for every point of the surface
- With modern hardware this is no big issue any more
  - Often even the normal is calculated per pixel
    - Bump or displacement maps

Problems in Interpolated Shading

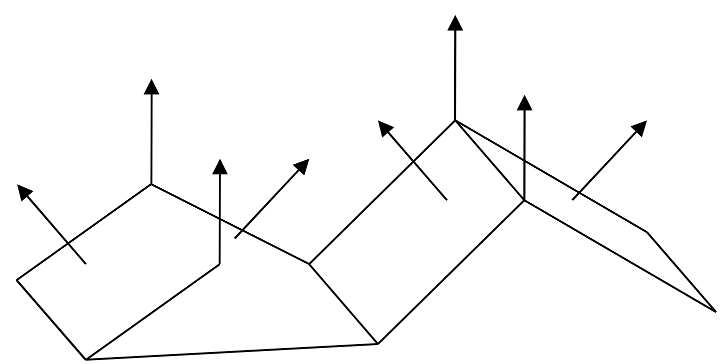
- **Problems**
  - Polygonal silhouette may not match the smooth shading
  - Perspective distortion
    - Interpolation may be performed after perspective transformation in the 2-D screen coordinate system, rather than world coordinate system.
  - Orientation dependence.
    - This problem does not concern triangles for which linear interpolation is rotation-invariant.
  - Shading discontinuities at shared vertices (T-edges).
  - Unrepresentative normal vectors.



Shading at **P** is interpolated along different scan-lines when polygon rotates.



T-edges



Vertex normals are all parallel