

Computer Graphics

- Texturing Methods -

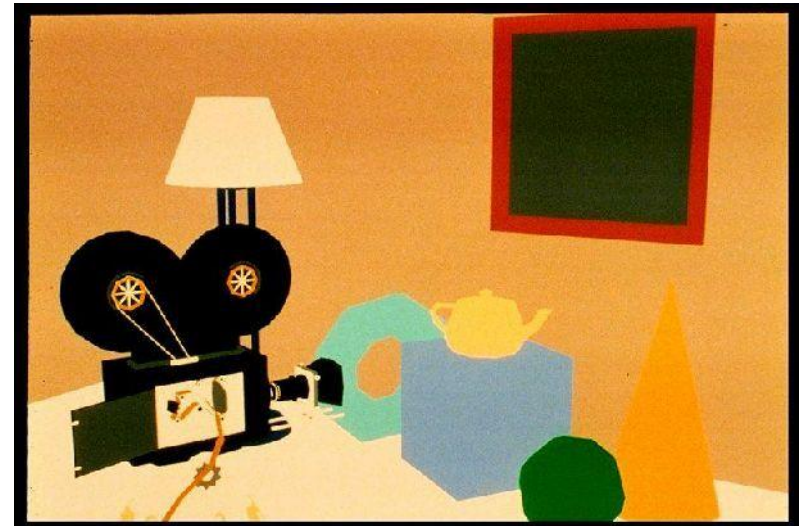
Overview

- **Last time**
 - BRDFs
 - Shading
- **Today**
 - Texturing
 - Texture parameterization
 - Procedural methods
 - Procedural textures
 - Fractal landscapes
- **Next lecture**
 - Texture filtering
 - Alias & signal processing

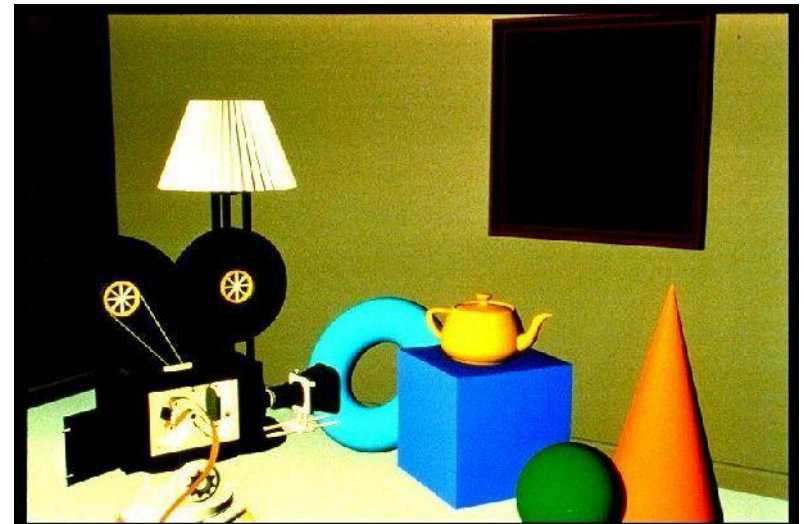
TEXTURING

Simple Illumination

- No illumination
- Constant colors

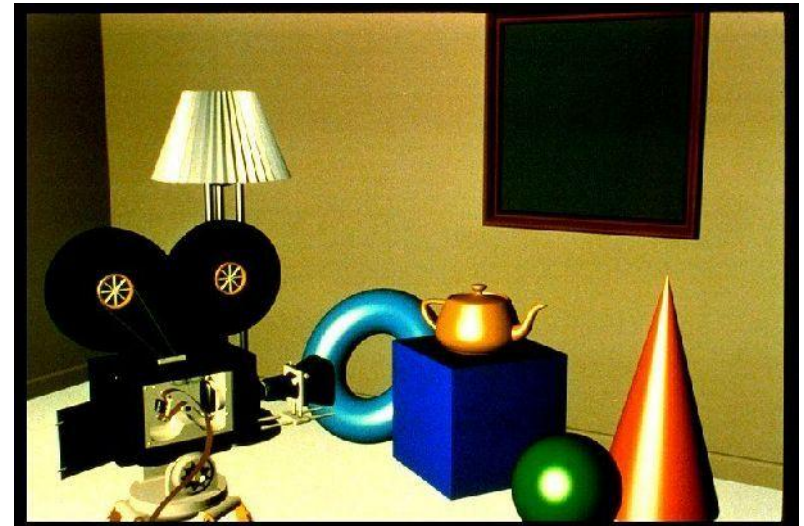


- Parallel light
- Diffuse reflection



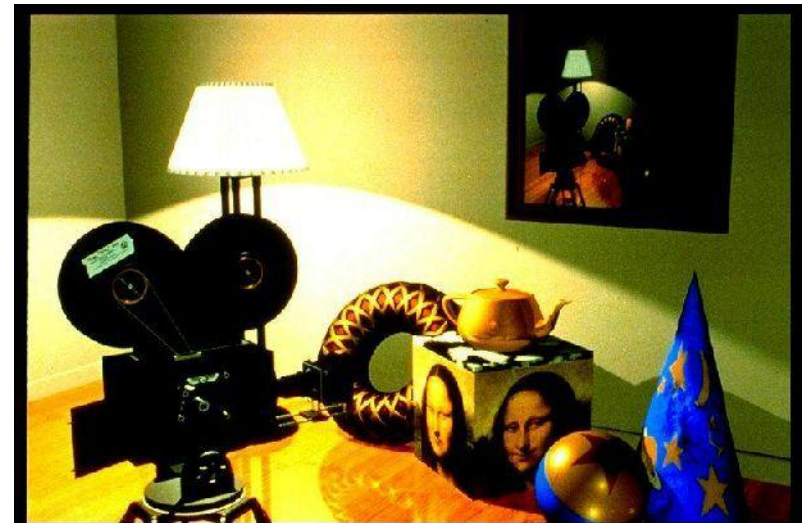
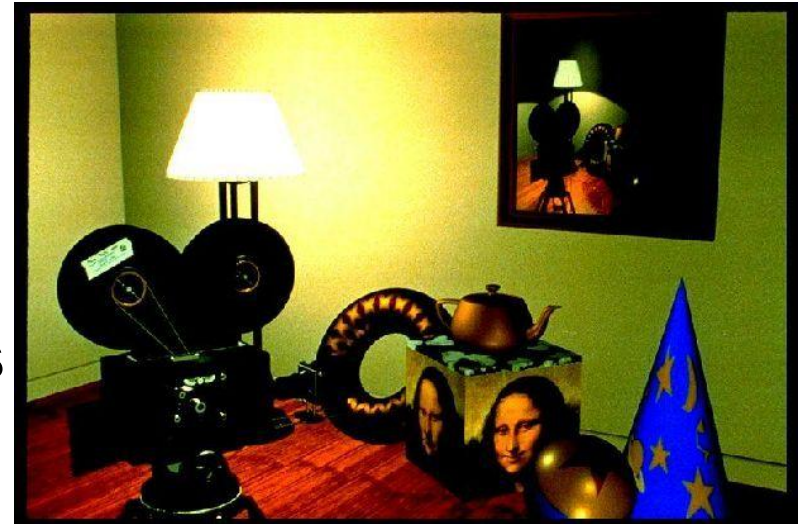
Standard Illumination

- Parallel light
- Specular reflection
- Multiple local light sources
- Different BRDFs
- **Object properties constant over surface**



Texturing

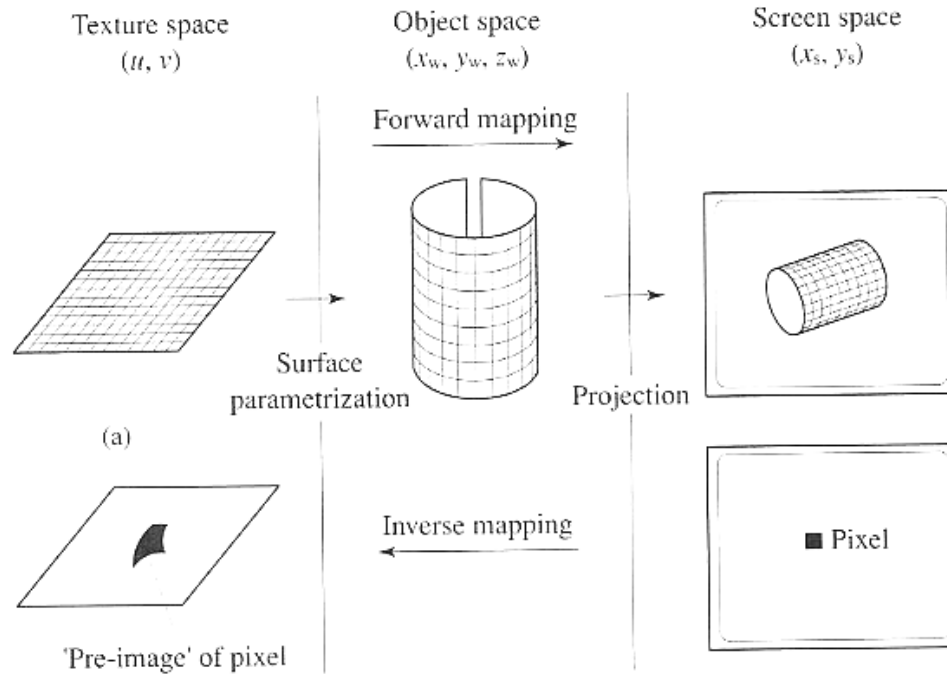
- **Varying object properties**
 - 2D image reflectance textures
 - Bump-mapping
- **Environment characteristics**
 - Shadows
 - Reflection textures



Texture-Modulated Quantities

- **Modulation of object surface properties**
- **Reflectance**
 - Diffuse reflection coefficient k_d
 - Specular reflection coefficient k_s
- **Opacity (α)**
 - Modulating transparency (e.g. for fences)
- **Normal vector**
 - Bump mapping: $N'(P) = N(P + t N)$ (in normal direction, height)
 - Normal mapping: $N' = N + \Delta N$ (arbitrary offset)
- **Geometry**
 - Displacement mapping: $P' = P + \Delta P$
- **Distant illumination**
 - Environment mapping / reflection mapping

2D Texture Mapping



- **Forward mapping**

- Object surface parameterization
- Projective transformation

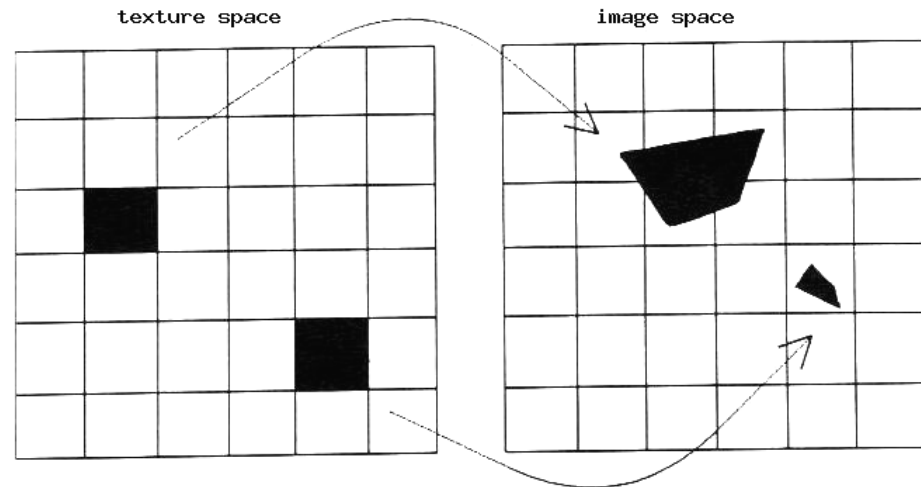
- **Inverse mapping**

- Find corresponding pre-image/footprint of each pixel in texture
- Integrate over pre-image

Forward Mapping

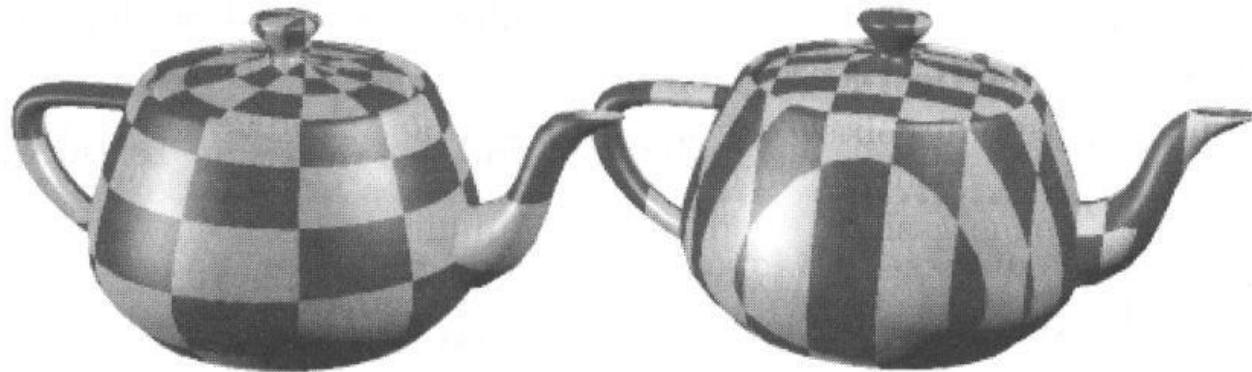
- **Maps each texel to its position in the image**
- **Uniform sampling of texture space does not guarantee uniform sampling in screen space**
 - Can create holes, need to scan-convert (see later)
- **Possibly used if**
 - The texture-to-screen mapping is difficult to invert
 - The texture image does not fit into memory
 - Process texture in tiles in order

- Texture scanning:
 - for v
 - for u
 - compute $x(u,v)$ and $y(u,v)$
 - copy $TEX[u,v]$ to $SCR[x,y]$



Surface Parameterization

- **To apply textures we need 2D coordinates on surfaces**
 - **Parameterization**
- **Some objects have a natural parameterization**
 - Sphere: spherical coordinates $(\varphi, \theta) = (2\pi u, \pi v)$
 - Cylinder: cylindrical coordinates $(\varphi, h) = (2\pi u, H v)$
 - Parametric surfaces (such as B-spline or Bezier surfaces → later)
- **Parameterization is less obvious for**
 - Polygons, implicit surfaces, teapots...



Triangle Parameterization

- **Triangle is a planar object**
 - Has implicit parameterization (e.g. barycentric coordinates)
 - But we need more control: placement of triangle in texture space
- **Assign texture coordinates (u,v) to each vertex (x_o, y_o, z_o)**
- **Apply viewing projection $(x_o, y_o, z_o) \rightarrow (x,y)$ (details later)**
- **Yields full texture transformation (warping) $(u,v) \rightarrow (x,y)$**

$$x = \frac{au + bv + c}{gu + hv + i} \qquad y = \frac{du + ev + f}{gu + hv + i}$$

- In homogeneous coordinates (by embedding (u,v) as $(u,v,1)$)

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix}; (x, y) = \left(\frac{x'}{w}, \frac{y'}{w} \right), (u, v) = \left(\frac{u'}{q}, \frac{v'}{q} \right)$$

- Transformation coefficients determined by 3 pairs $(u,v) \rightarrow (x,y)$
 - Three linear equations
 - Invertible iff neither set of points is collinear

Triangle Parameterization (2)

- **Given**
$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix}$$

- **The inverse transform $(x,y) \rightarrow (u,v)$ is**

$$\begin{bmatrix} u' \\ v' \\ q \end{bmatrix} = \begin{bmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{bmatrix} \begin{bmatrix} x' \\ y' \\ w \end{bmatrix}$$

- **Coefficients must be calculated for each triangle**

- Rasterization

- Incremental bilinear update of (u',v',q) in screen space
- Using the partial derivatives of the linear function (i.e. constants)

- Ray tracing

- Evaluated at every intersection

- **Often derivatives are needed as well**

- Explicitly given in matrix

Cylinder Parameterization

- Transformation from texture space to the cylinder parametric representation can be written as:

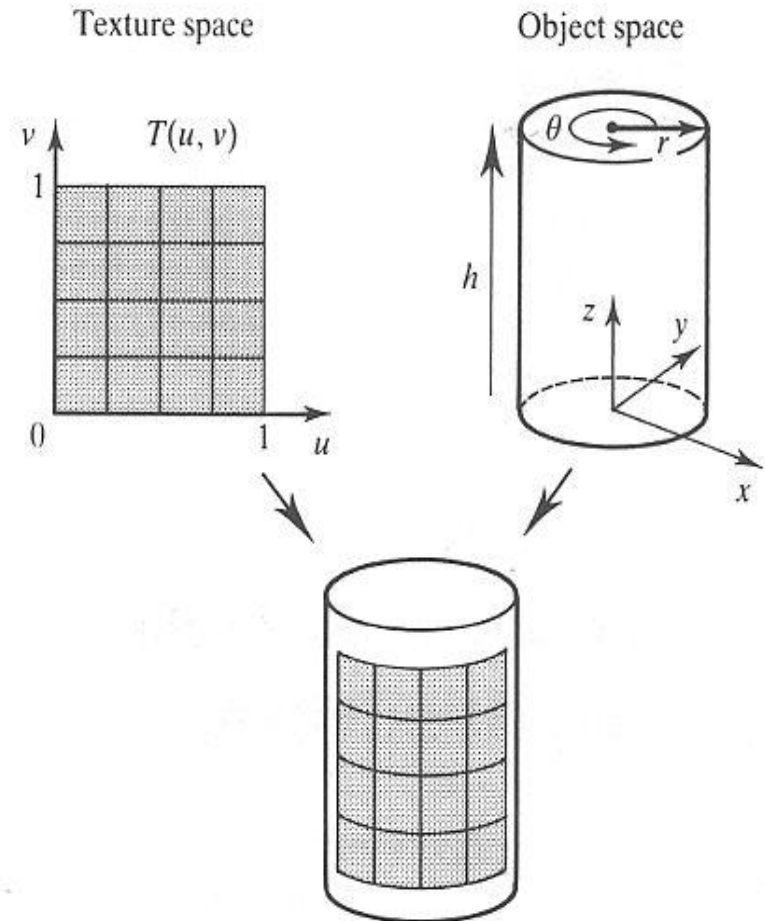
$$(\theta, h) = (2\pi u, vH)$$

- where H is the height of the cylinder.
- The surface coordinates in the Cartesian reference frame can be uniquely expressed as:

$$x_o = r \cos \theta$$

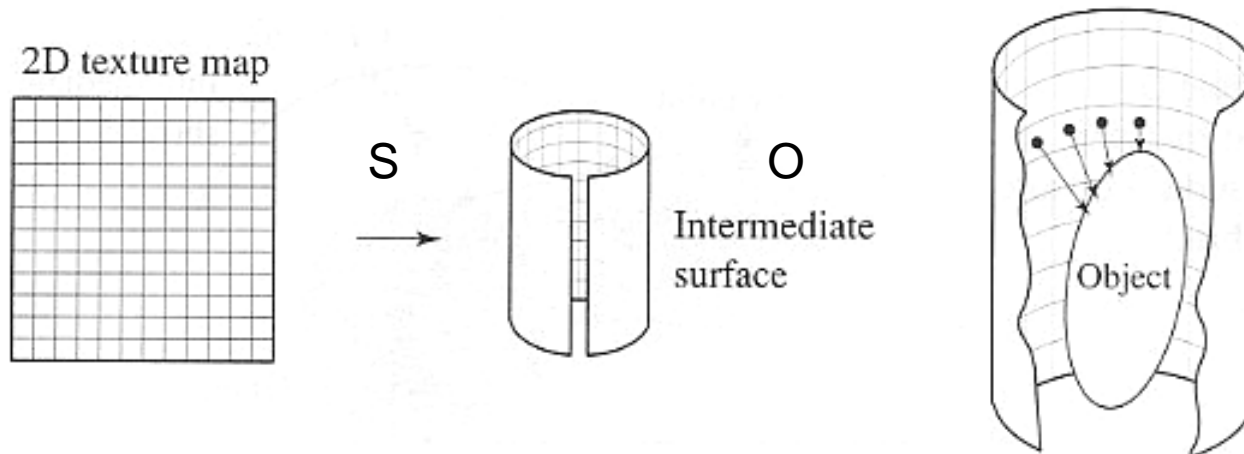
$$y_o = r \sin \theta$$

$$z_o = h$$



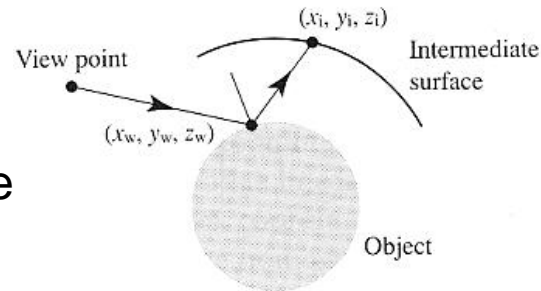
Two-Stage Mapping

- **Inverse mapping for arbitrary 3D surfaces too complex**
- **Approximation technique is used:**
 - Mapping from 2D texture space to a simple 3D intermediate surface (S mapping)
 - Should be a reasonable approximation of the destination surface
 - E.g.: plane, cylinder, sphere, cube, ...
- **Mapping from the intermediate surface to the destination object surface (O mapping)**

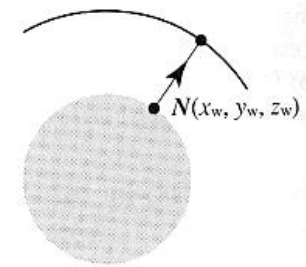


O-Mapping

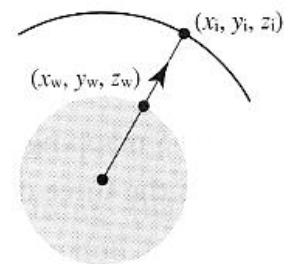
- **Determine point on intermediate surface through**
 - Reflected view ray
 - Reflection or environment mapping
 - Normal mapping
 - Line through object centroid
 - Shrink-wrapping
 - Forward mapping
 - Normal mapping from intermediate surface



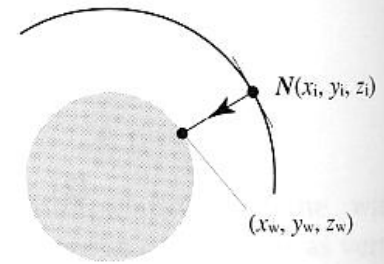
(1) Reflected ray



(2) Object normal



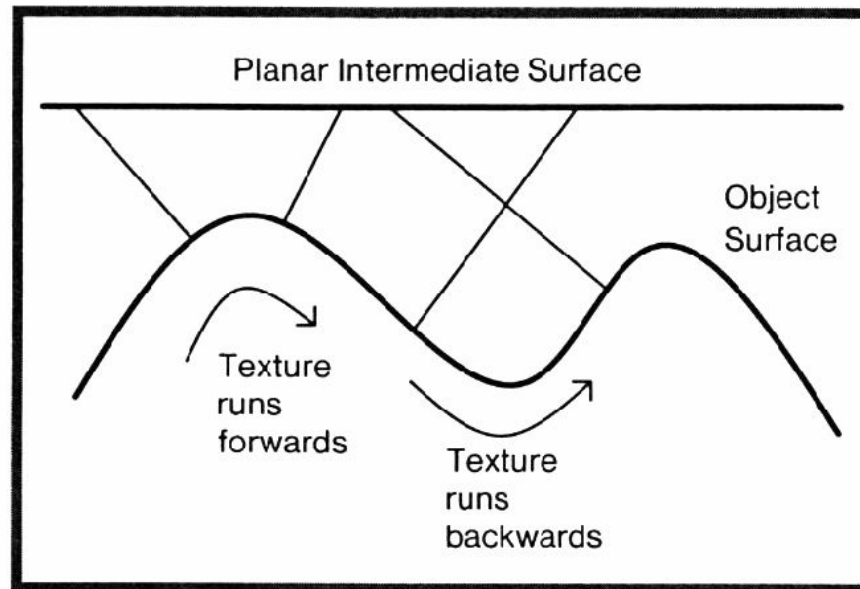
(3) Object centroid



(4) Intermediate surface normal

Two-Stage Mapping: Problems

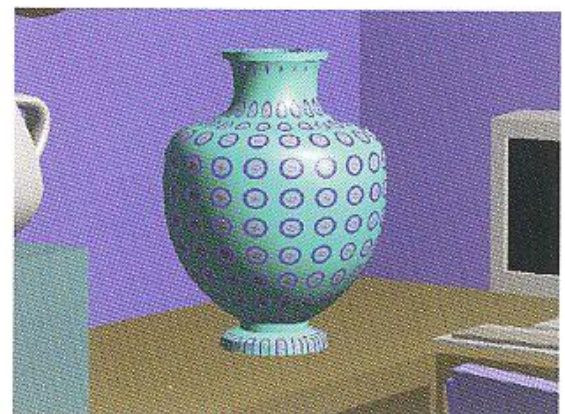
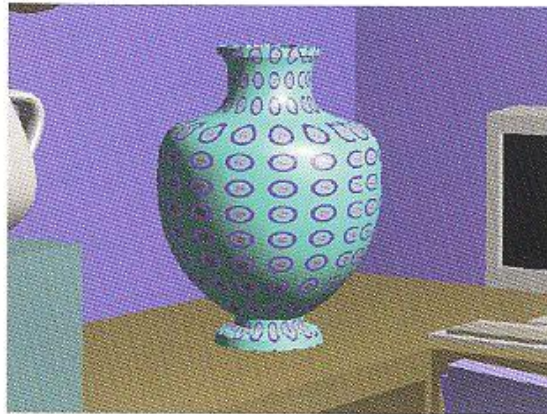
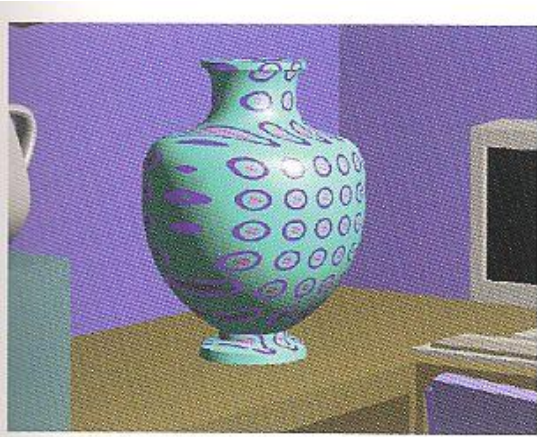
- **May introduce undesired texture distortions if the intermediate surface differs too much from the destination surface**
- **Still often used in practice because of its simplicity**



Surface concavities can cause the texture pattern to reverse if the object normal mapping is used.

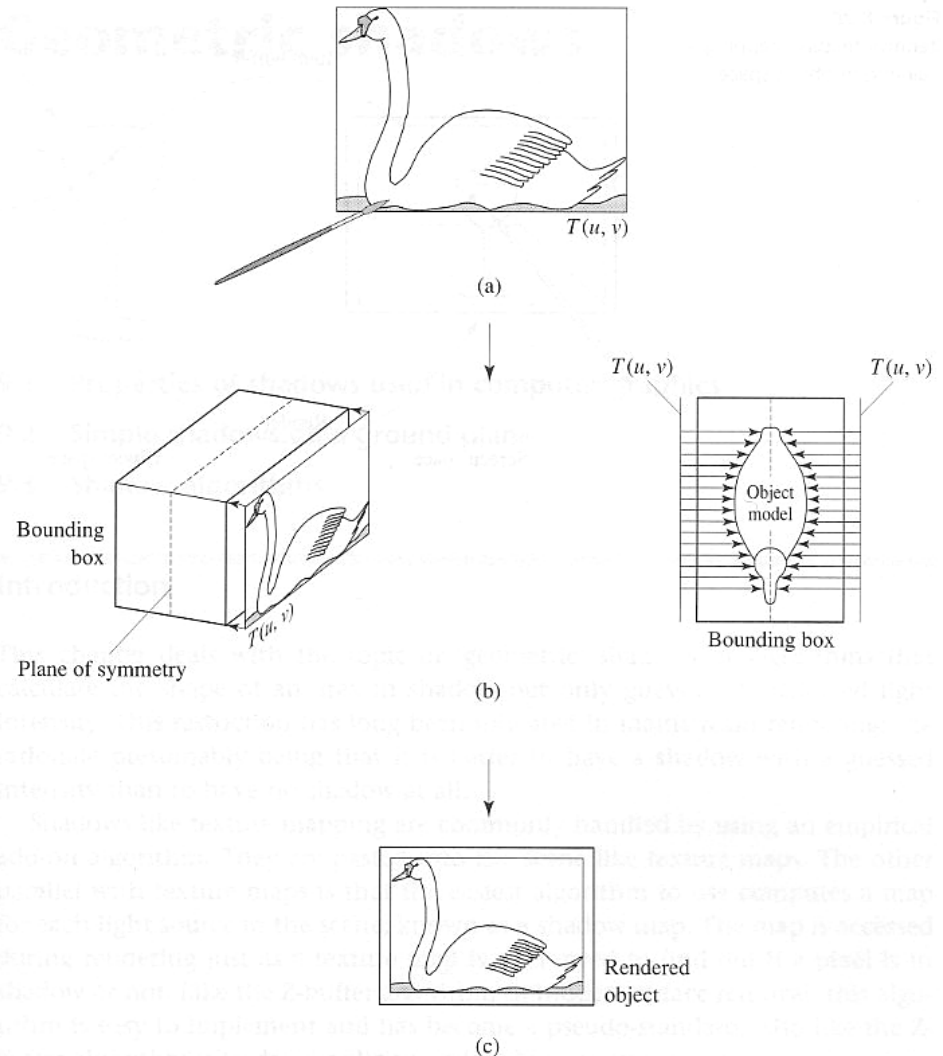
Two-Stage Mapping: Example

- **Different intermediate surfaces**
- **Plane**
 - Strong distortion where object surface normal \perp to plane normal
- **Cylinder**
 - Reasonably uniform mapping (symmetry !)
- **Sphere**
 - Problems with concave regions

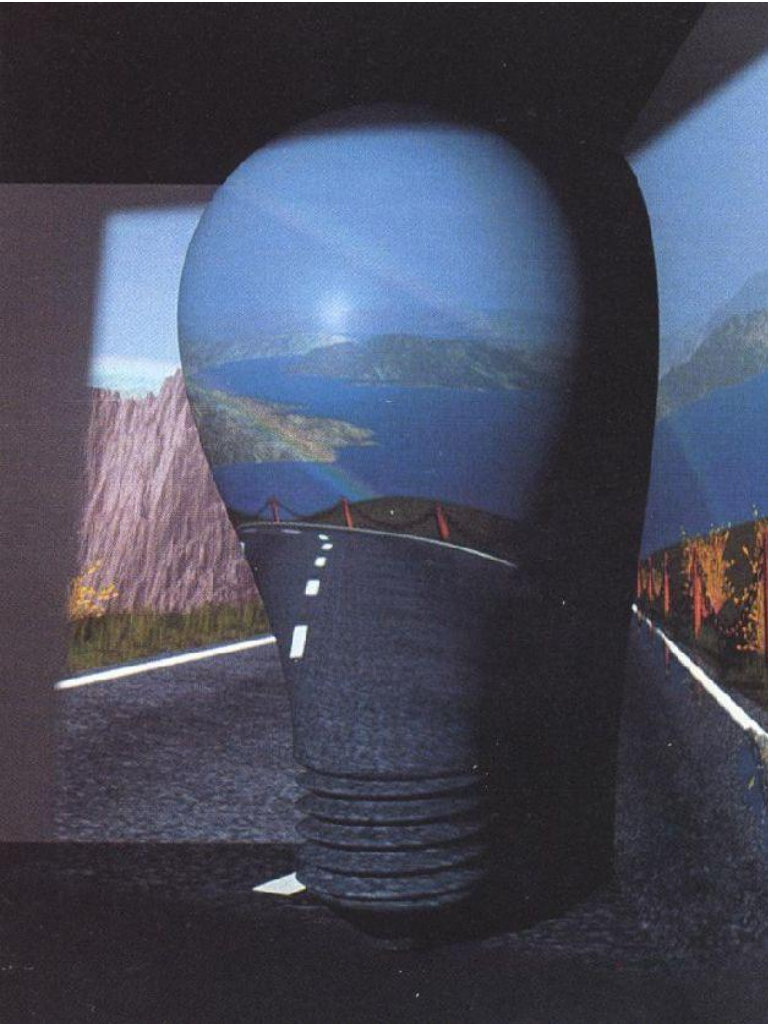


Projective Textures

- **Project texture onto object surfaces**
 - Slide projector
- **Parallel or perspective projection**
- **Use photographs as textures**
- **Multiple images**
 - View-dependent texturing (advanced topic)
- **Perspective mapping**

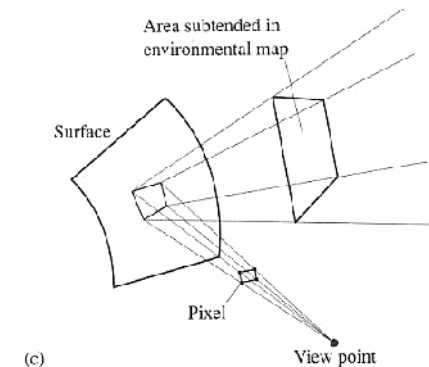
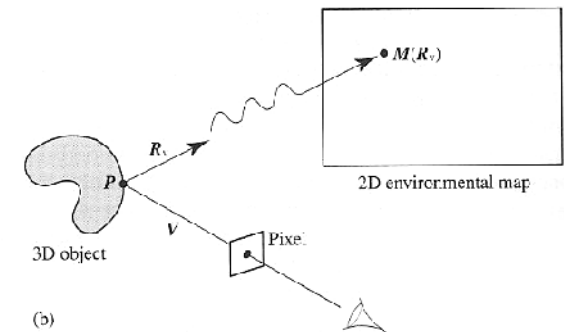
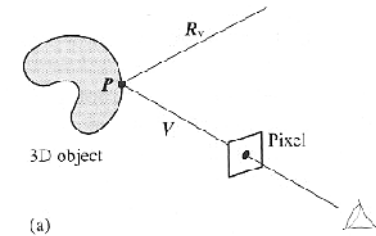


Projective Texturing: Examples



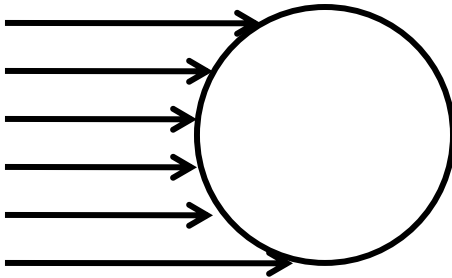
Reflection Mapping

- Also called “environment mapping”
- Reflection map parameterization
 - Intermediate surface in 2-stage mapping
 - Often cube, sphere, or double paraboloid
- Assumption: Distant illumination
 - Parallax-free illumination
 - No self-reflections, distortion of near objects
- Option: Separate map per object
 - Often necessary to be reasonably accurate
 - Reflections of other objects
 - Maps must be recomputed after changes
- Mirror reflections
 - Surface curvature: beam tracing
 - Map filtering



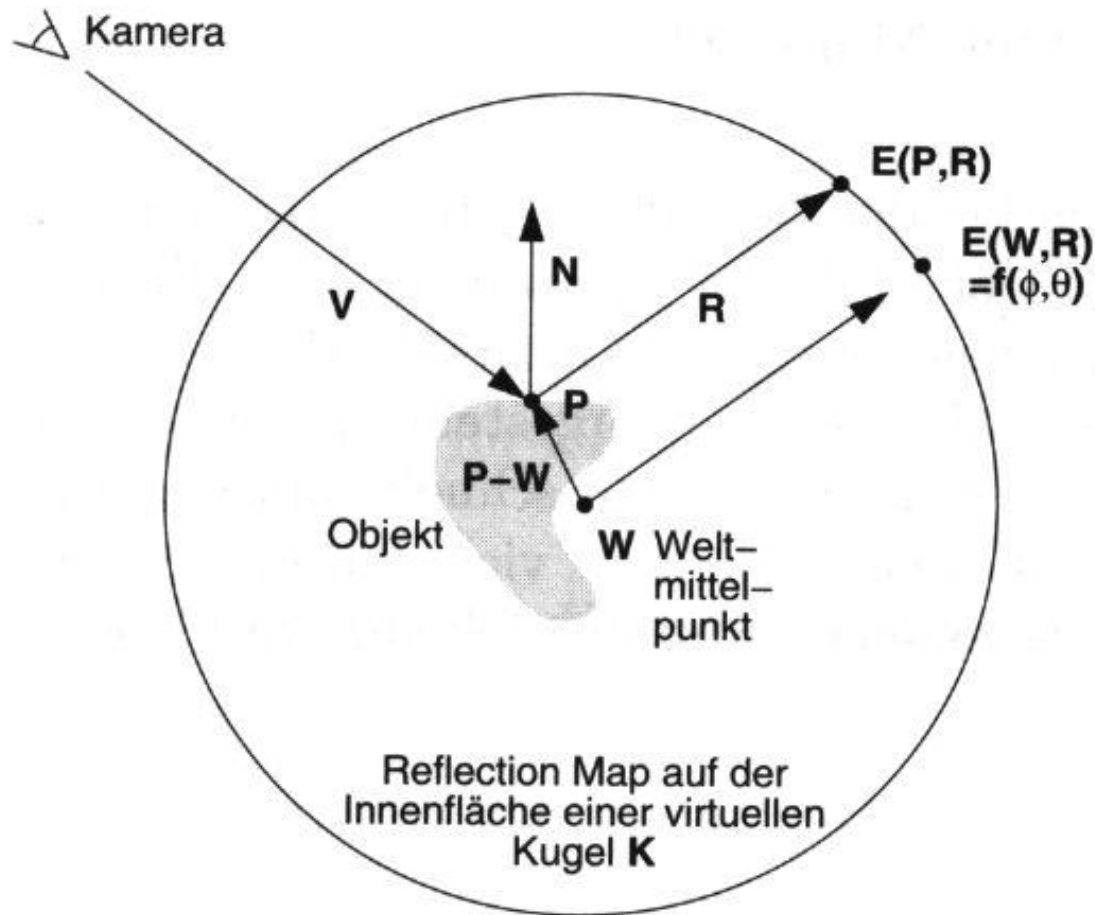
Reflection Map Acquisition

- **Generating spherical maps (original 1982/83)**
 - I.e. photo of a reflecting sphere (gazing ball)



Reflection Map Rendering

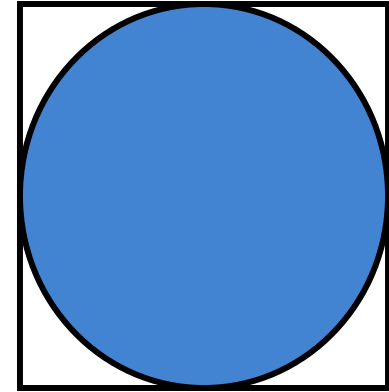
- Spherical parameterization
- O-mapping using reflected view ray intersection



Reflection Map Parameterization

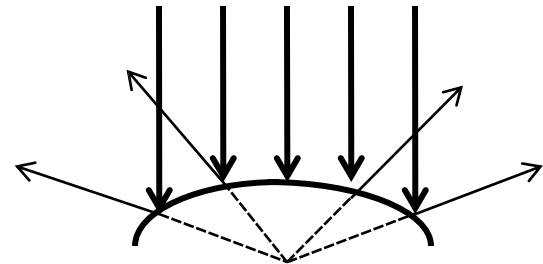
- **Spherical mapping**

- Single image
- Bad utilization of the image area
- Bad scanning on the edge
- Artifacts, if map and image do not have the same view point



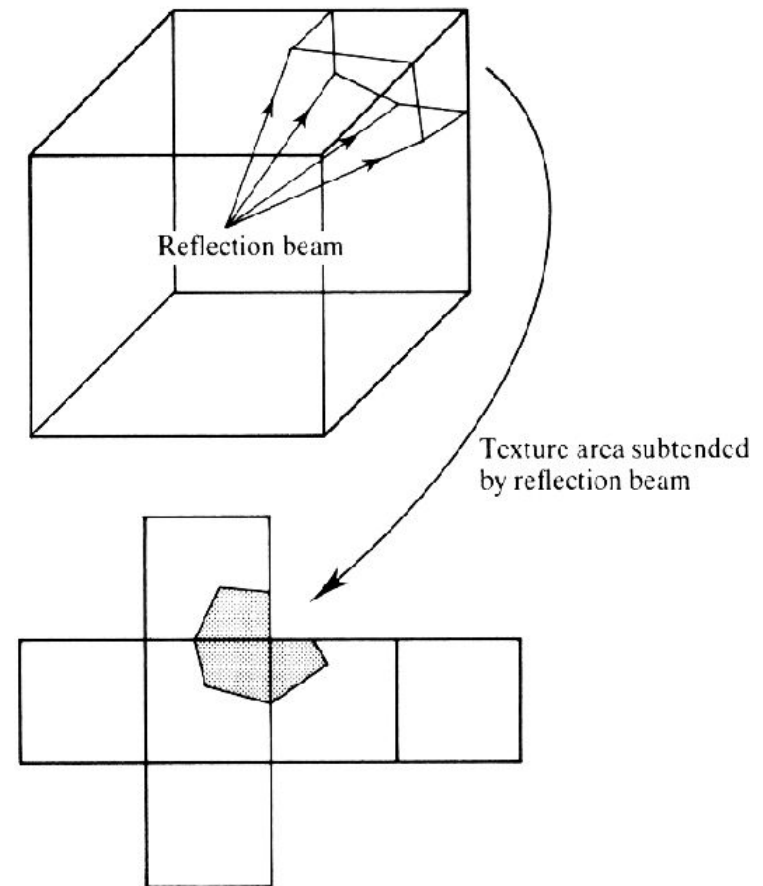
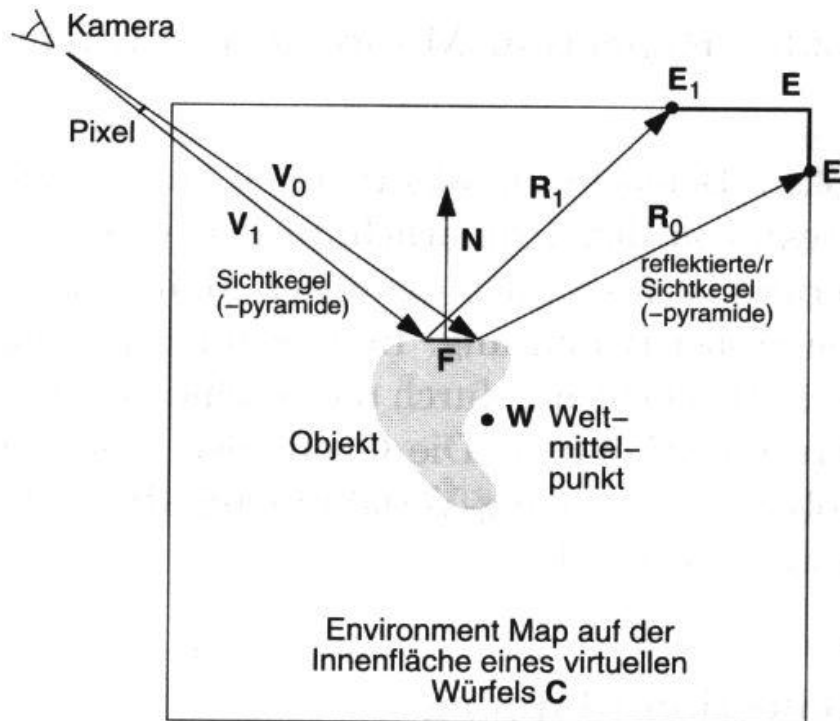
- **Double parabolic mapping**

- Yields spherical parameterization
- Subdivide in 2 images (front-facing and back-facing sides)
- Less bias near the periphery
- Arbitrarily reusable
- Supported by OpenGL extensions



Reflection Map Parameterization

- **Cubical environment map, cube map, box map**
 - Enclose object in cube
 - Images on faces are easy to compute
 - Poorer filtering at edges
 - Support in OpenGL



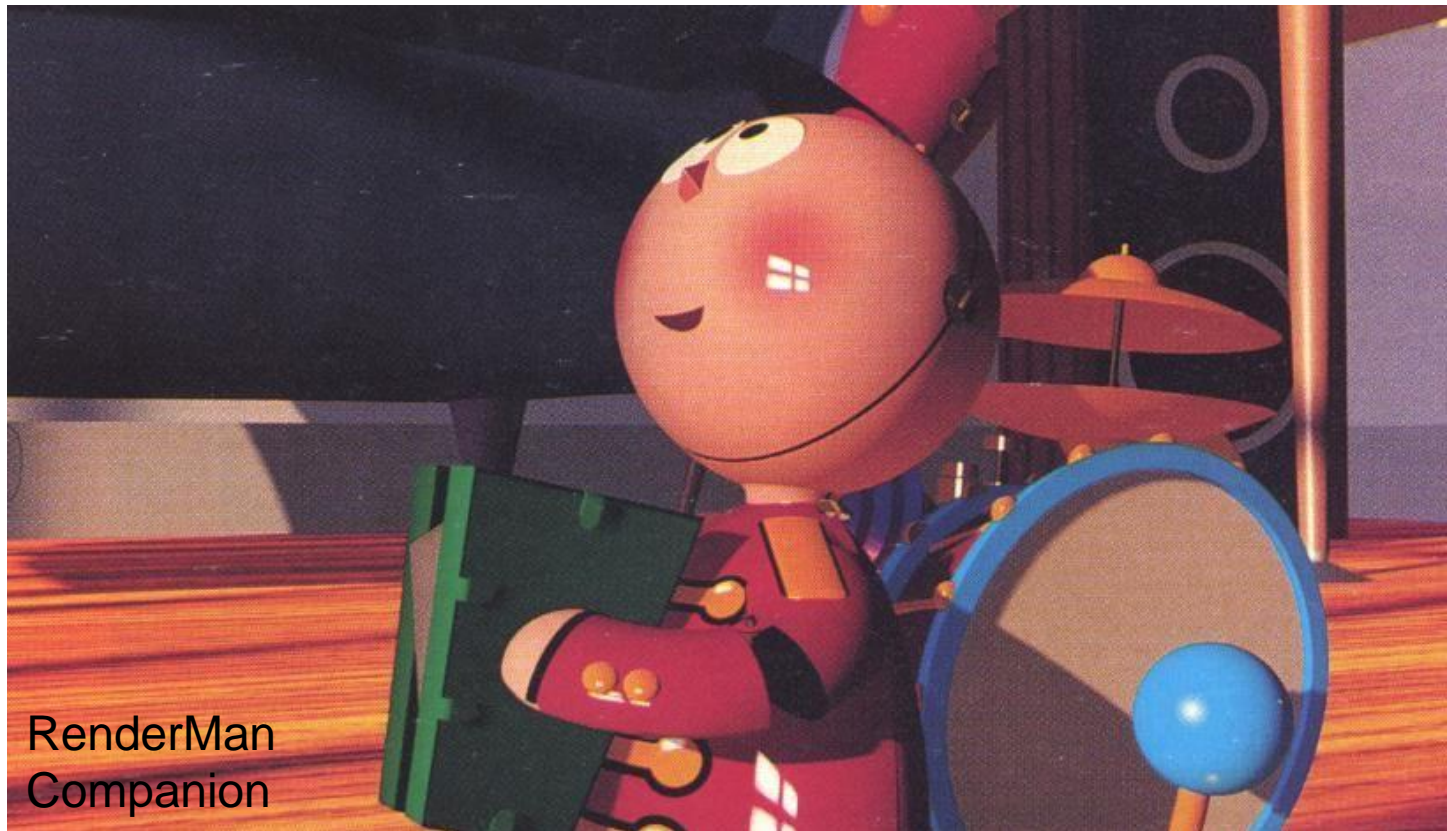
Reflection Mapping Example



Terminator II motion picture

Reflection Mapping Example II

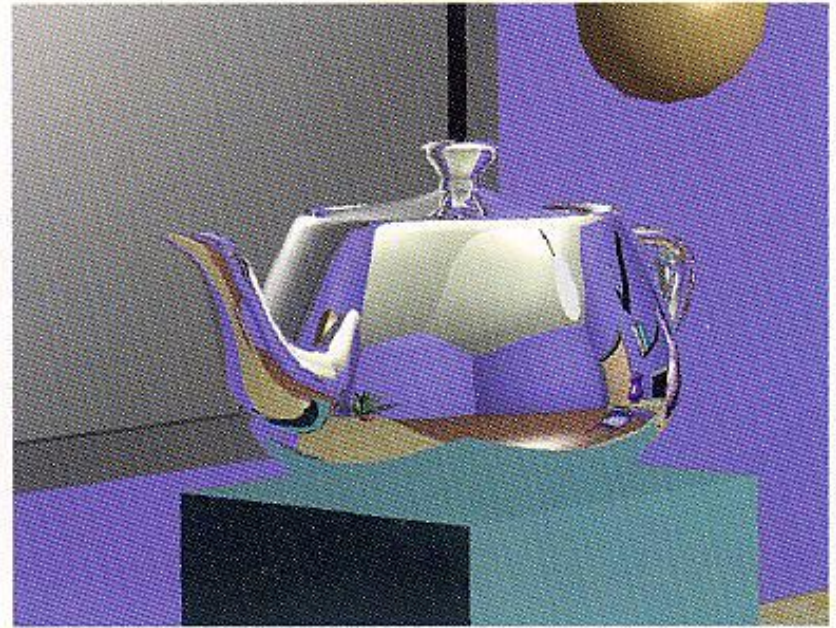
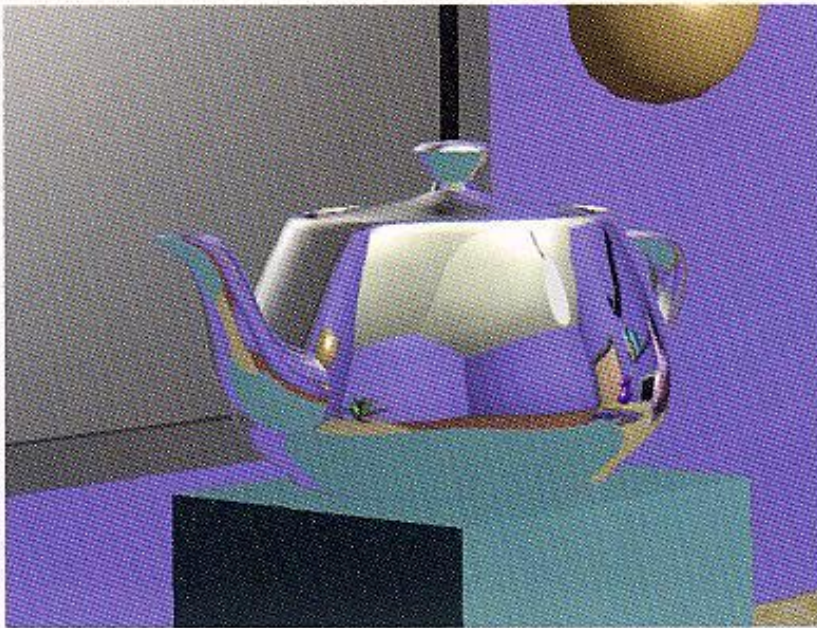
- **Reflection mapping with Phong reflection**
 - Two maps: diffuse & specular
 - Diffuse: index by surface normal
 - Specular: indexed by reflected view vector



RenderMan
Companion

Ray Tracing vs. Reflection Map

- Differences ?



Recursive Ray Tracing

- How to fake it with reflection mapping?

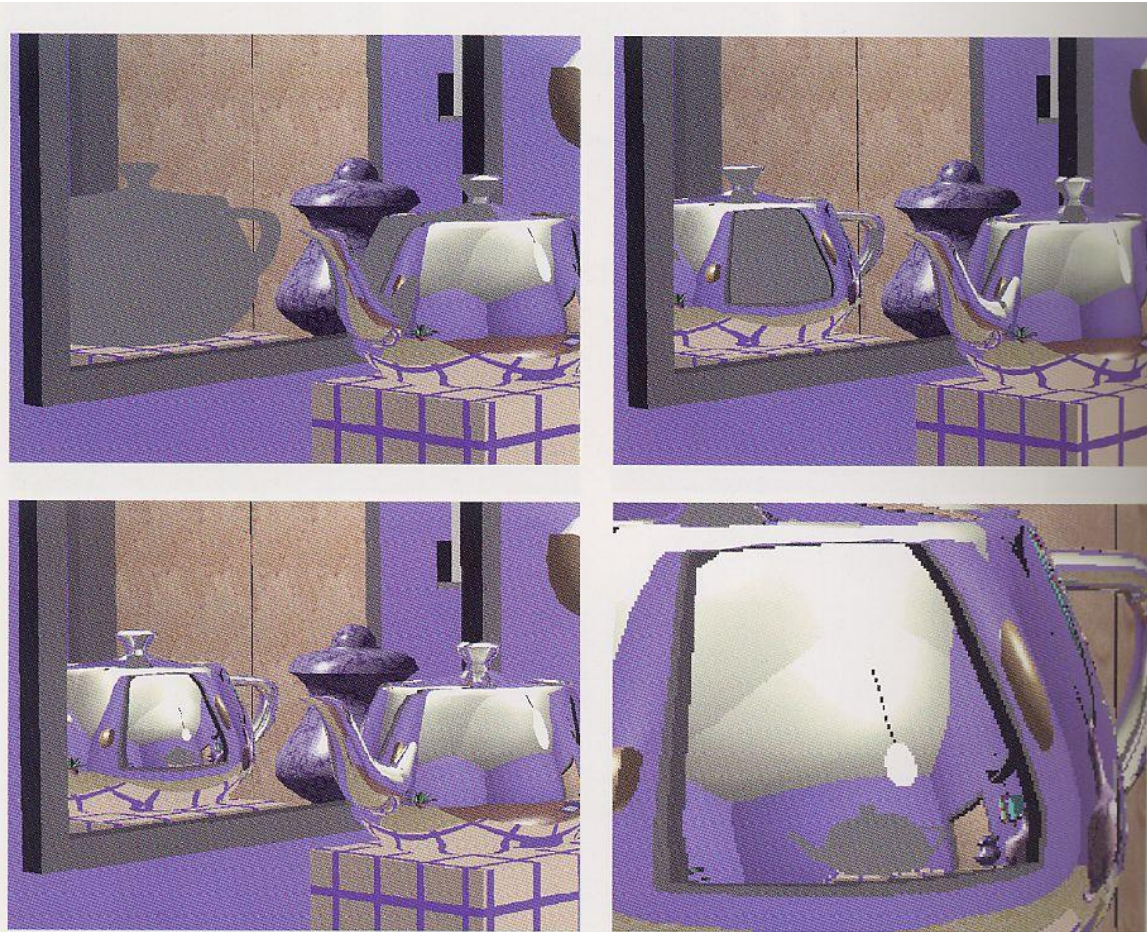
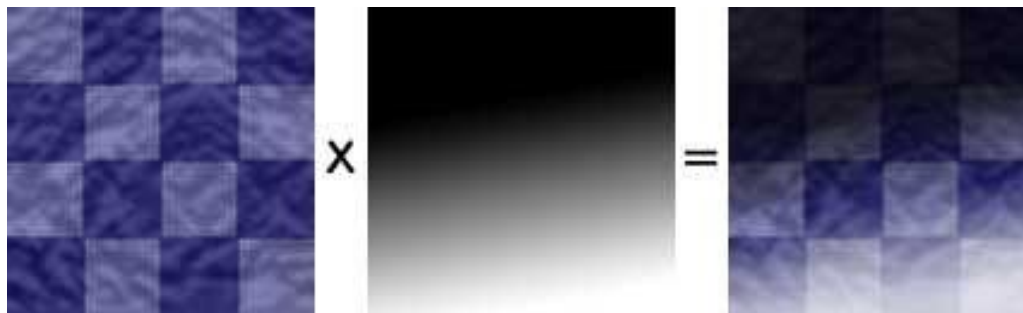


Figure 18.11

A recursive depth demonstration. The trace terminates at depth 2, 3, 4 and 5 (zoom image) respectively. 'Unassigned' pixels are coloured grey. Bad aliasing as a function of recursive depth (the light cable) is apparent.

Light Maps

- **Light maps (e.g. in Quake)**
 - Pre-calculated illumination (local irradiance)
 - Often very low resolution: smoothly varying
 - Multiplication of irradiance with base texture
 - Diffuse reflectance only
 - Provides surface radiosity
 - View-independent out-going radiance
 - Animated light maps
 - Animated shadows, moving light spots, etc...

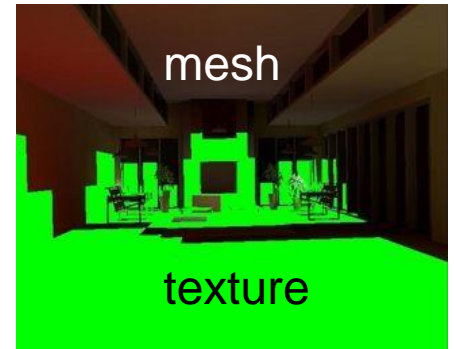
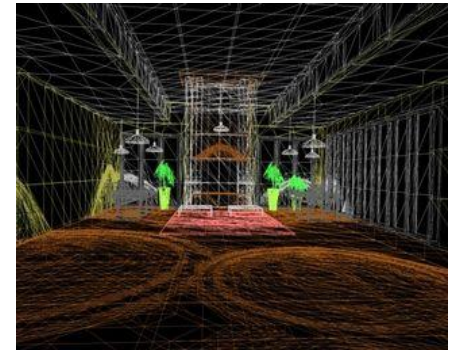


Reflectance

Irradiance

Radiosity

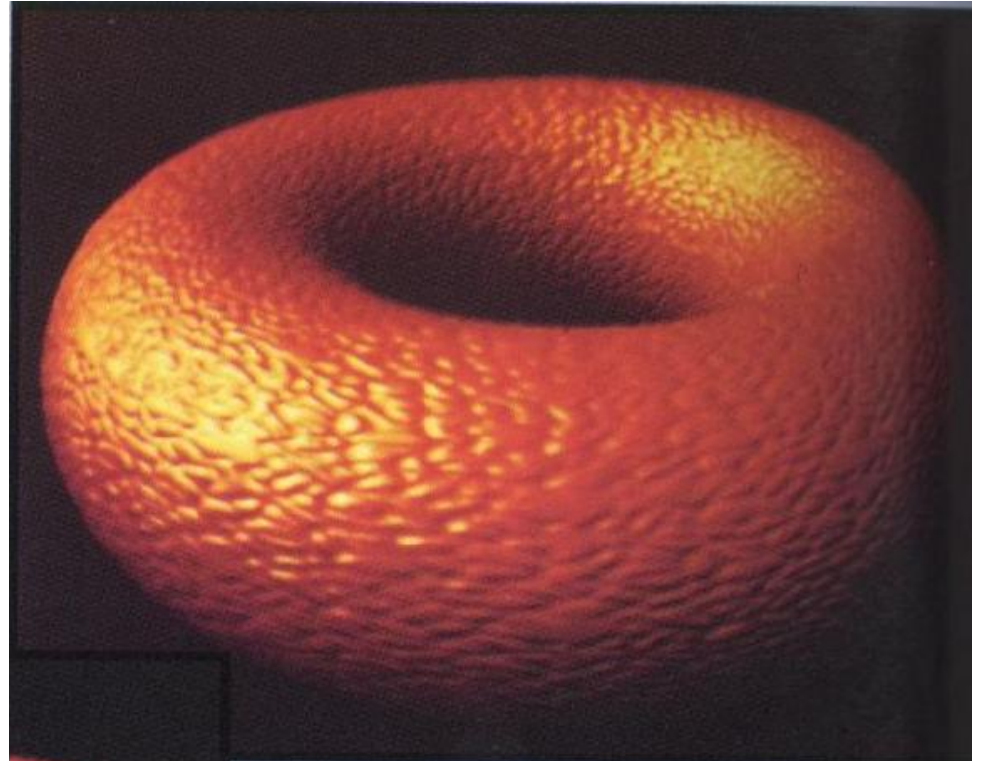
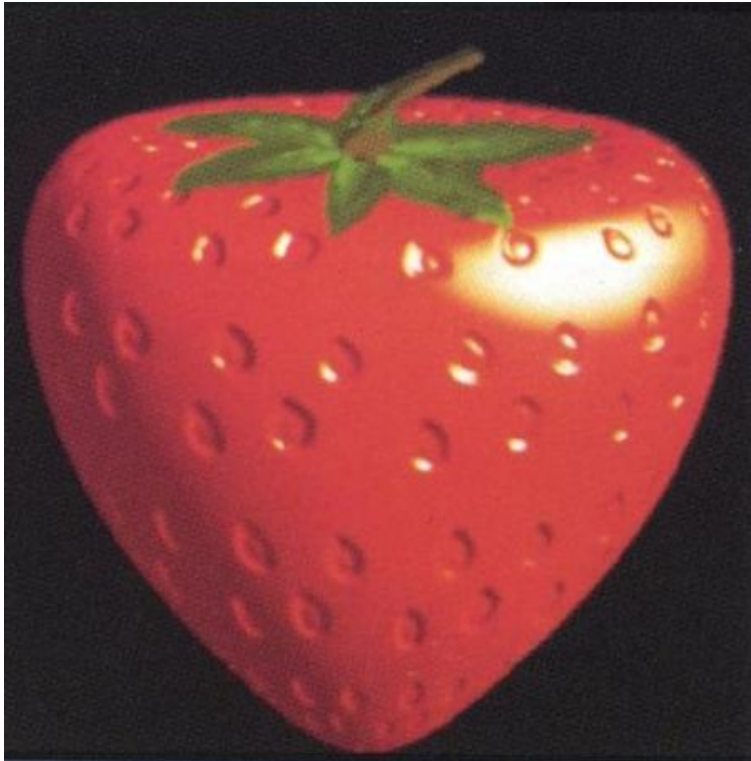
$$B(x) = \rho(x) E(x) = \pi L_o(x)$$



Representing radiosity in a mesh or texture

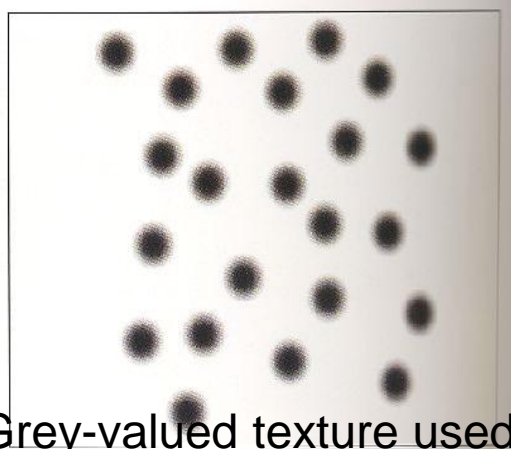
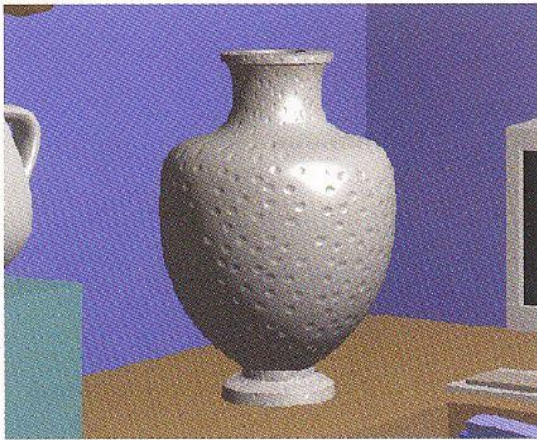
Bump Mapping

- **Modulation of the normal vector**
 - Surface normals changed only
 - Influences shading only
 - No self-shadowing, contour is **not** altered

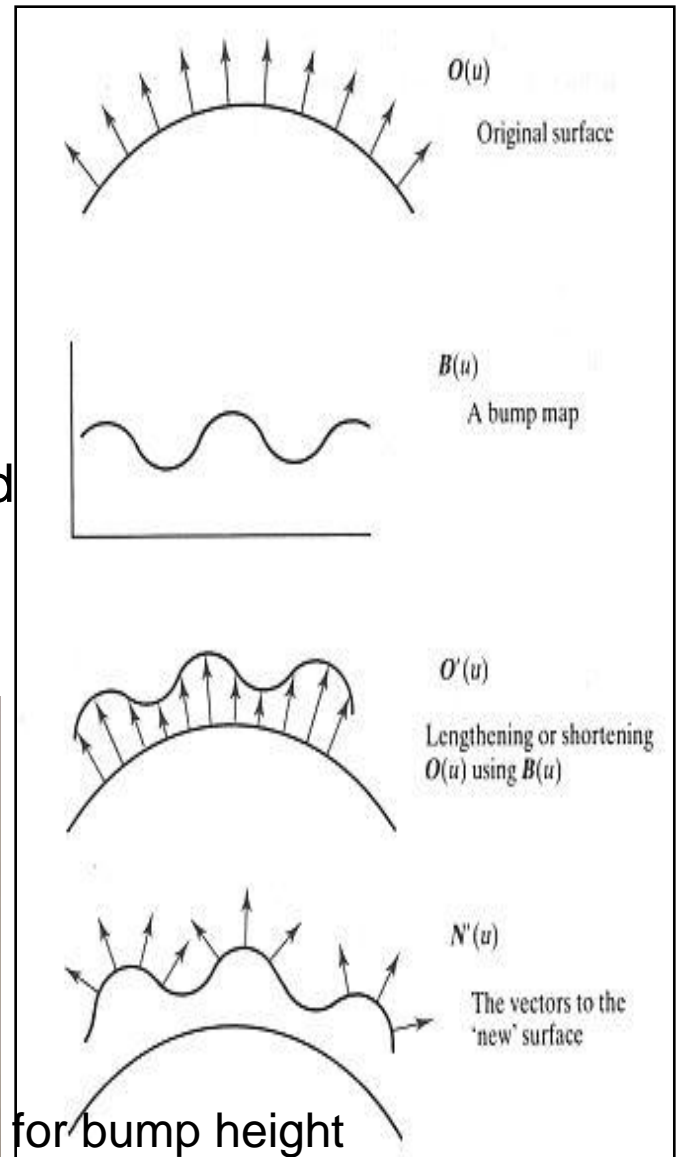


Bump Mapping

- **Original surface:** $O(u, v)$
 - Surface normals are known
- **Bump map:** $B(u, v) \in R$
 - Surface is offset in normal direction according to bump map intensity
 - New normal directions $N'(u, v)$ are calculated based on virtually displaced surface $O'(u, v)$
 - Original surface is rendered with new normals $N'(u, v)$



Grey-valued texture used for bump height



Bump Mapping

$$O'(u, v) = O(u, v) + B(u, v) \frac{N}{|N|}$$

- Normal is cross-product of derivatives:

$$O'_u = O_u + B_u \frac{N}{|N|} + B \left(\frac{N}{|N|} \right)_u$$

$$O'_v = O_v + B_v \frac{N}{|N|} + B \left(\frac{N}{|N|} \right)_v$$

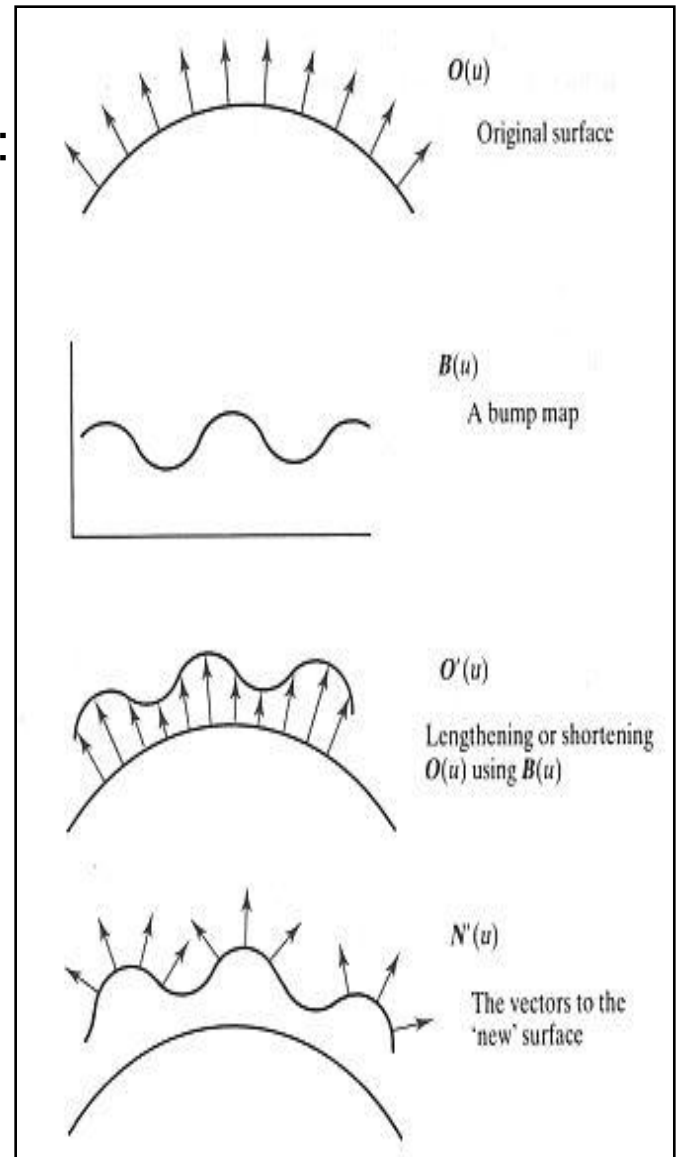
- If B is small (i.e. the bump map displacement function is small compared to its spatial extent) the last term in each equation can be ignored

$$N'(u, v) = O_u \times O_v + B_u \left(\frac{N}{|N|} \times O_v \right) + B_v \left(O_u \times \frac{N}{|N|} \right) + B_u B_v \left(\frac{N \times N}{|N|^2} \right)$$

- The first term is the normal to the surface and the last is zero, giving:

$$D = B_u(N \times O_v) - B_v(N \times O_u)$$

$$N' = N + D$$

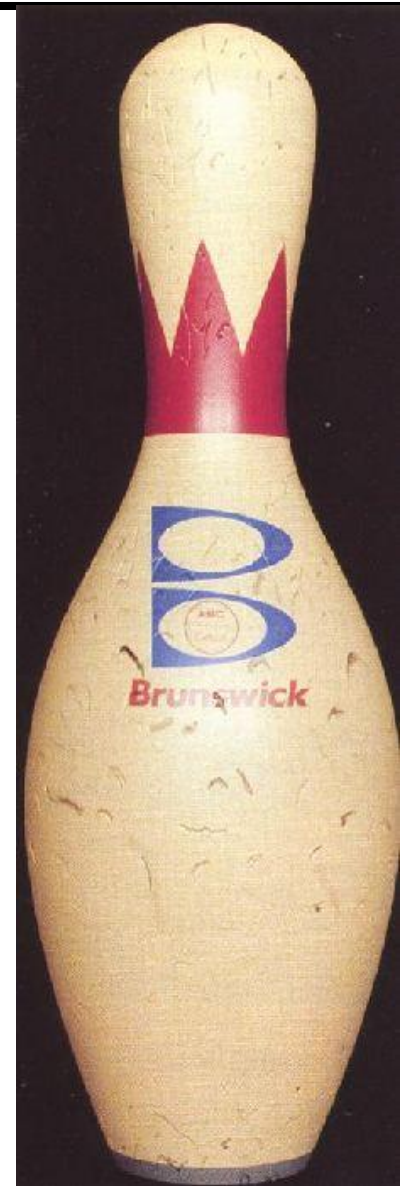


Texture Examples

- **Complex optical effects**
 - Combination of multiple texture effects

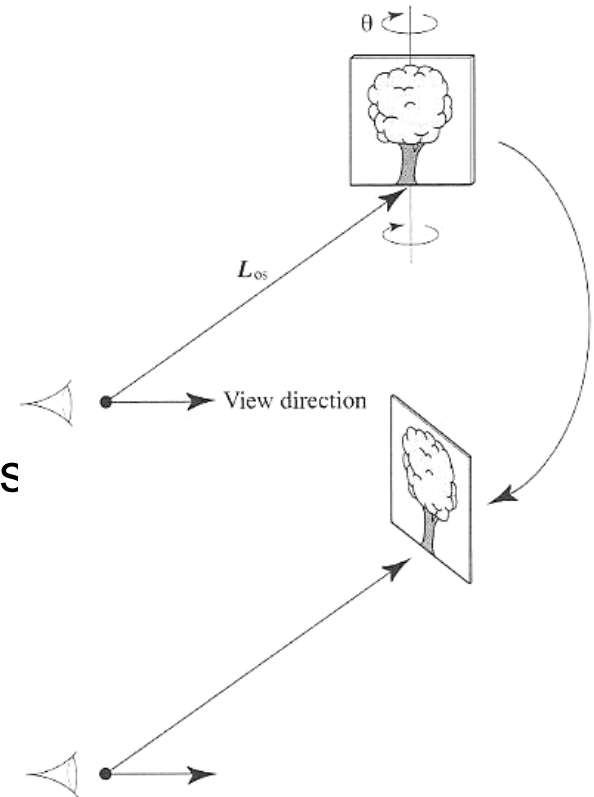
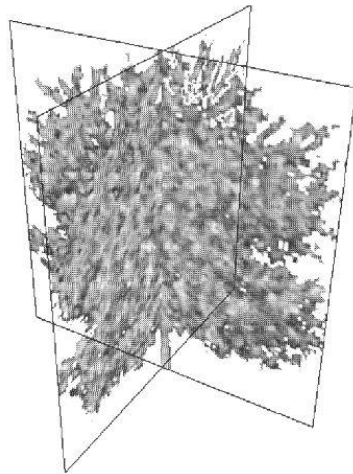


RenderMan Companion



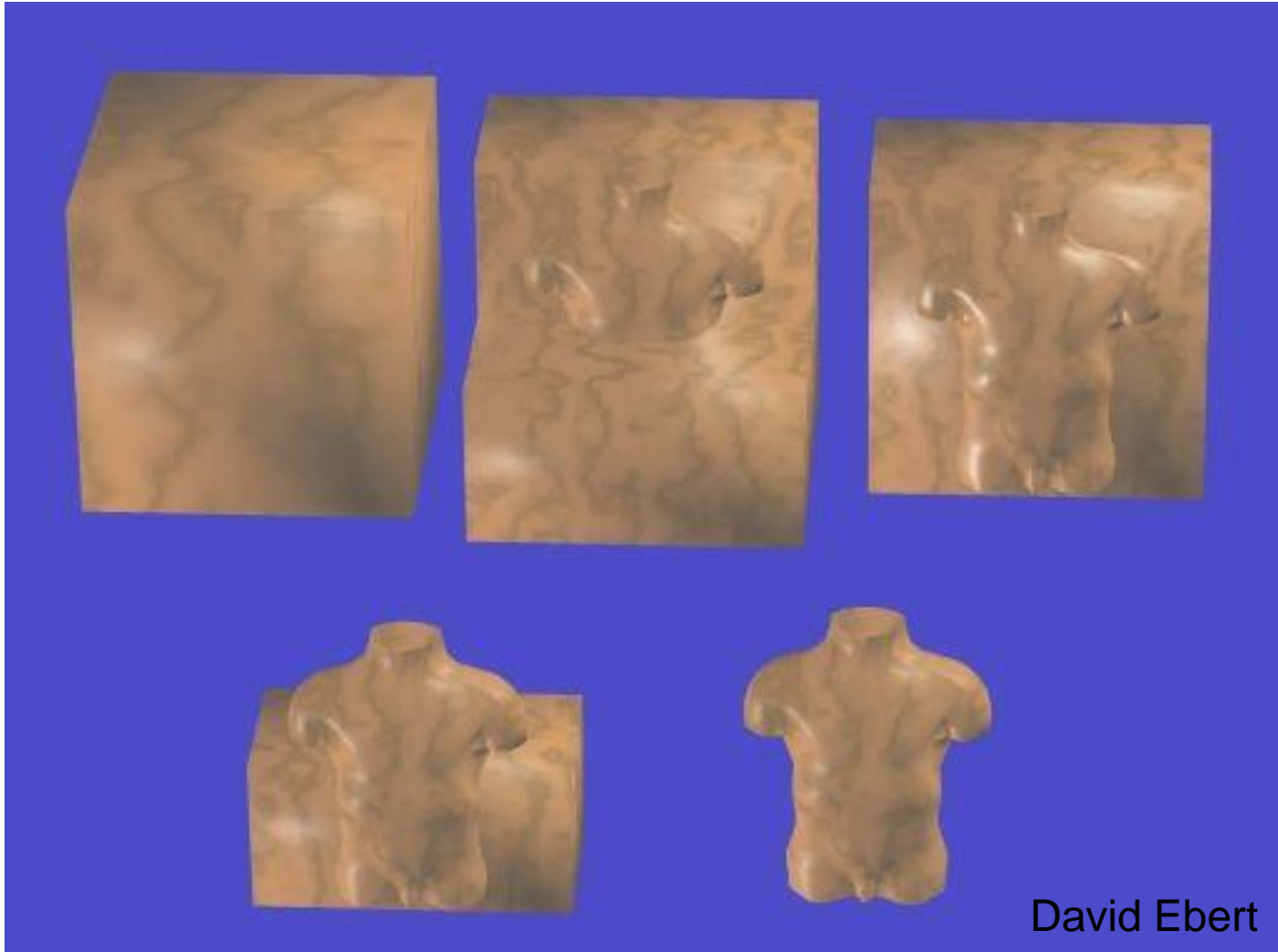
Billboards

- **Single textured polygons**
 - Often with opacity texture
 - Rotates, always facing viewer
 - Used for rendering distant objects
 - Best results if approximately radially or spherically symmetric
- **Multiple textured polygons**
 - Azimuthal orientation: different view-points
 - Complex distribution: trunk, branches, ...



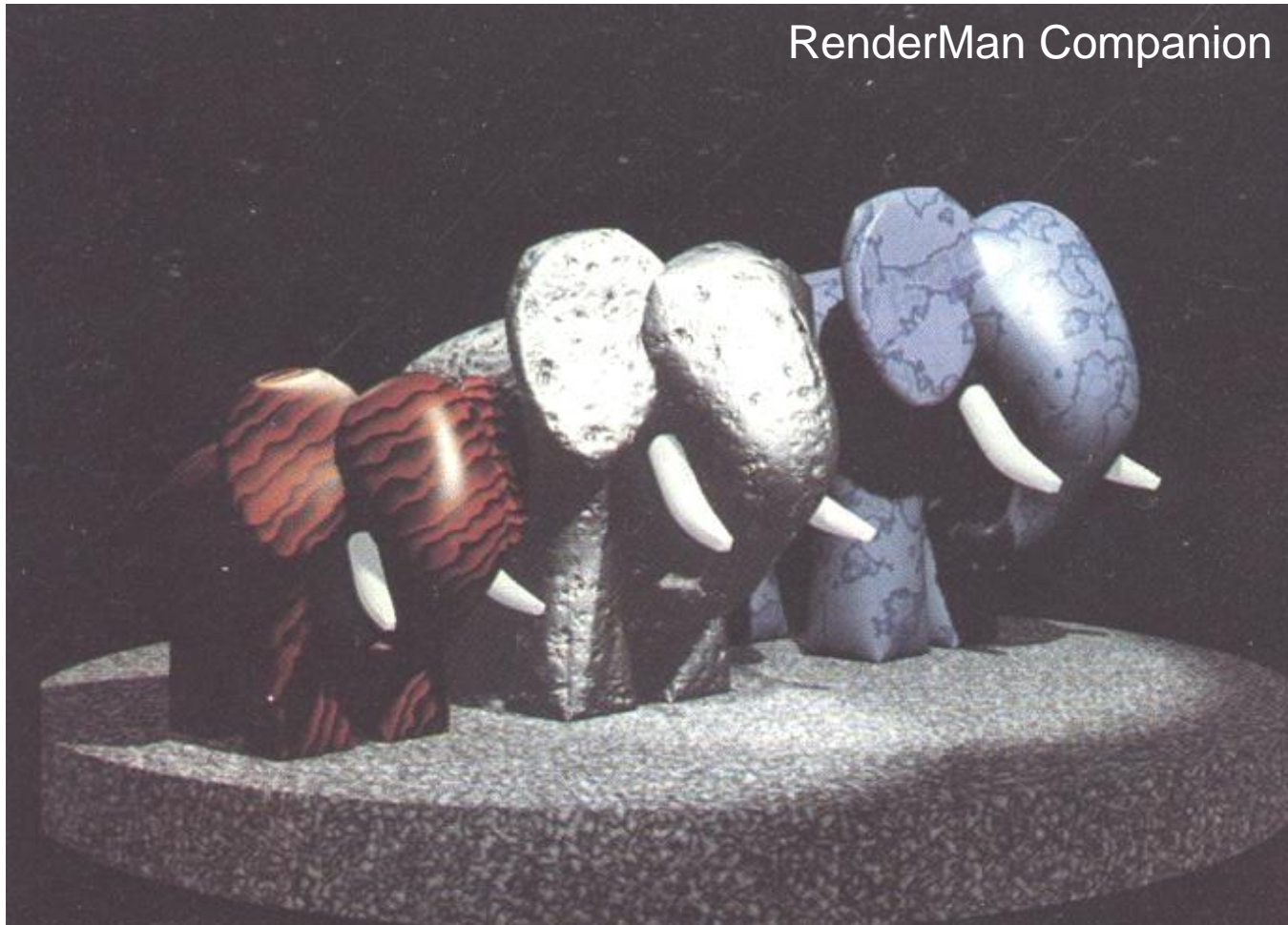
3-D Textures

- “Carving object shape out of material block”



Texture Examples

- **Solid 3D textures (wood, marble)**
- **Bump map (middle)**



Part II

Procedural Methods

Texture Maps | Procedural Textures

- **Texture maps: paintings, photos, videos, simulation...**
 - Simple acquisition
 - Illumination “frozen” during acquisition
 - Limited resolution, aliasing
 - High memory requirements
 - Mapping issues
- **Procedural textures**
 - Non-trivial programming
 - Flexibility & parametric control
 - Unlimited resolution
 - Anti-aliasing possible
 - Low memory requirements
 - Low-cost visual complexity
 - Can adapt to arbitrary geometry

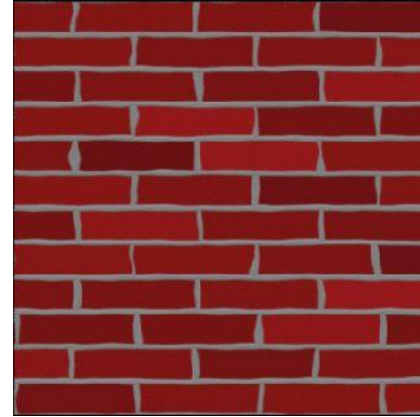
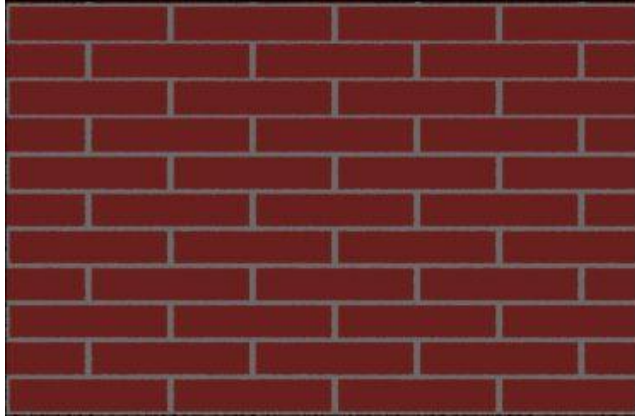


Procedural Textures

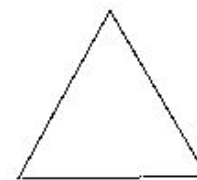
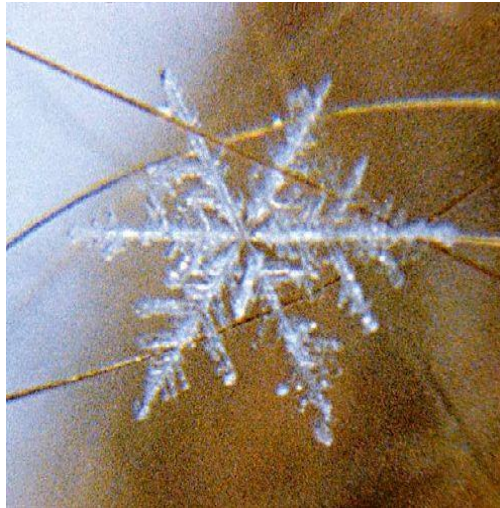
- **Function of some shading parameter**
 - E.g. world space, texture coordinates, ...
- **Texturing: evaluation of function on object surface**
 - Ray tracing: at intersection point with surface
 - Must be able to evaluate at random position efficiently
- **Observation: textures of natural objects**
 - Similarity between patches at different locations
 - Repetitiveness, coherence (e.g. skin of a tiger or zebra)
 - Similarity on different resolution scales
 - Self-similarity
 - But never completely identical
 - Additional disturbances, turbulence, noise
- **Goal: generic procedural texture function**
 - Mimics statistical properties of natural textures
 - Purely empirical approach
 - Looks convincing, but has nothing to do with material's physics

Texture Examples

- **Translational similarity**



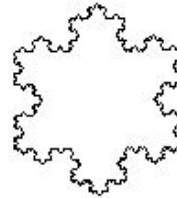
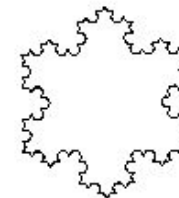
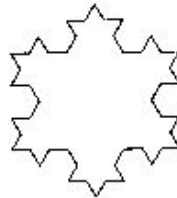
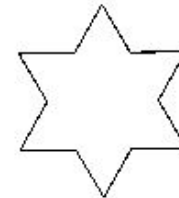
- **Similarity on different scales**



initiator

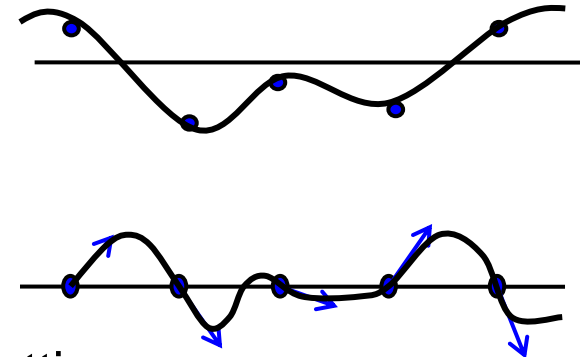


generator



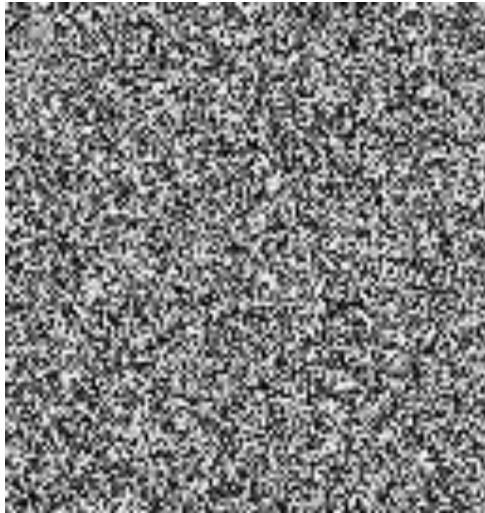
3D / Solid Noise: Perlin Noise

- **Noise(x,y,z)**
 - Statistical invariance under rotation
 - Statistical invariance under translation
 - Roughly one specific frequency
- **Integer lattice (i,j,k)**
 - Fixed fundamental frequency of ~ 1 Hz over lattice
 - Don't store all values – use a hash function to randomize and look up from a fixed-size table
 - **Value noise**: Random value at lattice
 - **Gradient noise**: Random gradient vector at lattice point Q: $G(Q)$
 - Value at point P: $G \cdot (P-Q)$
 - Tri-linear interpolation or cubic interpolation
 - Hermite spline \rightarrow later
- **Unlimited domain due to lattice and hashing**
- **Also see**
 - <http://www.noisemachine.com/talk1/>
 - <http://www.cs.cmu.edu/~mzucker/code/perlin-noise-math-faq.html>

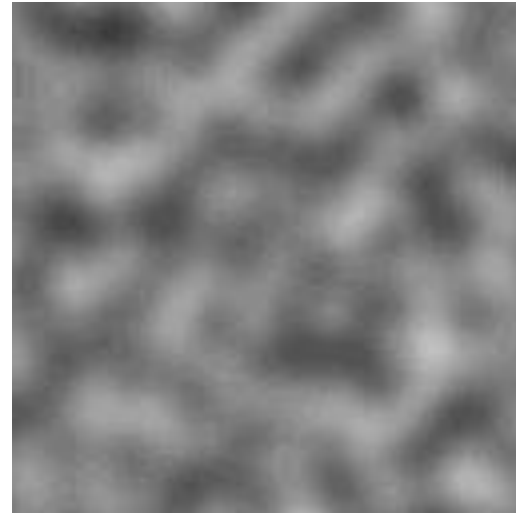


Noise vs. Noise

- **Gradient noise better than value noise**
 - Less regularity artifacts
 - More high frequencies in noise spectrum
 - Even tri-linear interpolation produces good results
- **Comparison between random values and Perlin noise**



Random values
at each pixel



Gradient noise

Turbulence Function

- **Noise function**

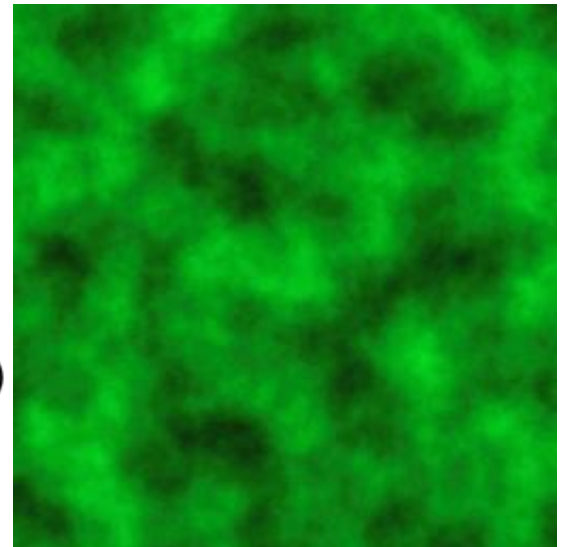
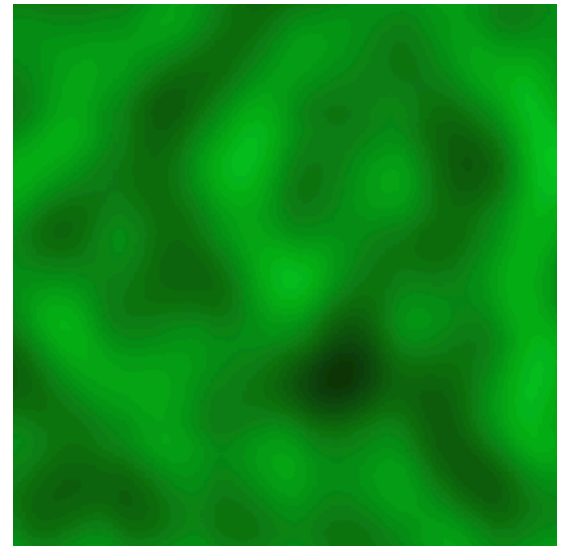
- Single spike in frequency spectrum

- **Natural textures**

- Decreasing power spectrum towards high frequencies

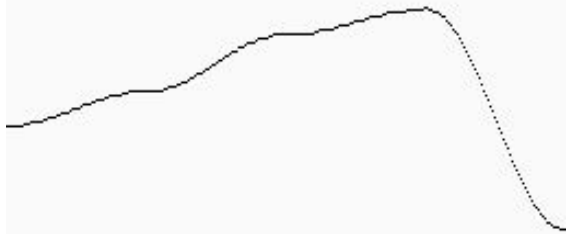
- **Turbulence from noise**

- $Turbulence(x) = \sum_{i=0}^k |a_i * noise(f_i x)|$
 - Frequency: $f_i = 2^i$
 - Amplitude: $a_i = 1 / p^i$
 - Persistence: p typically $p=2$
- Summation truncation
 - 1st term: $noise(x)$
 - 2nd term: $noise(2x)/2$
 - ...
 - Until period $(1/f_k) < 2$ pixel-size (band limit)
- Power spectrum : $a_i = 1 / f_i$
- Brownian motion: $a_i = 1 / f_i^2$

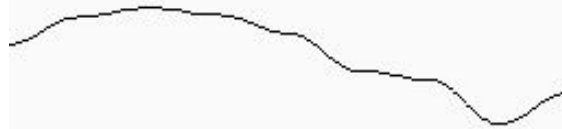


Synthesis of Turbulence (1D)

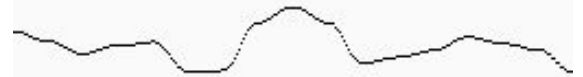
Amplitude : 128
frequency : 4



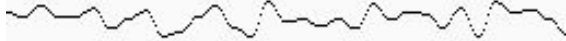
Amplitude : 64
frequency : 8



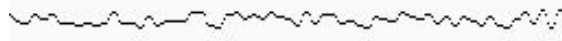
Amplitude : 32
frequency : 16



Amplitude : 16
frequency : 32



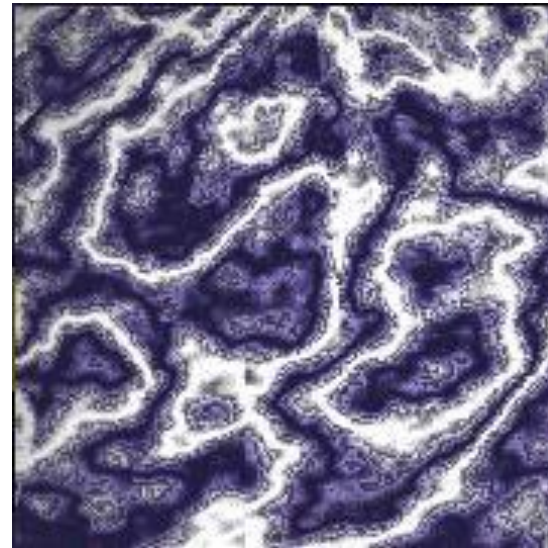
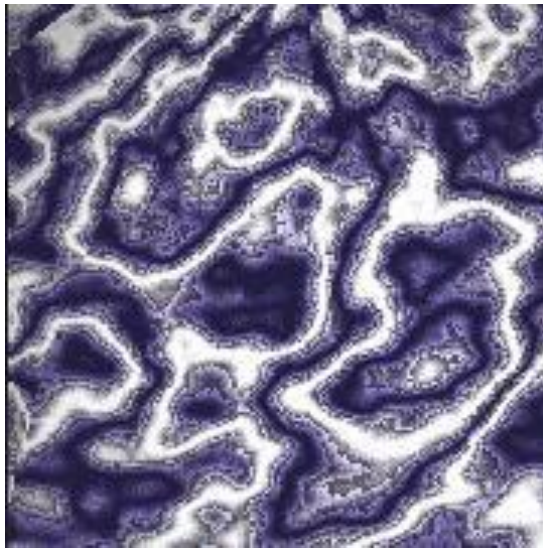
Amplitude : 8
frequency : 64



Sum of Noise Functions = (Perlin Noise)

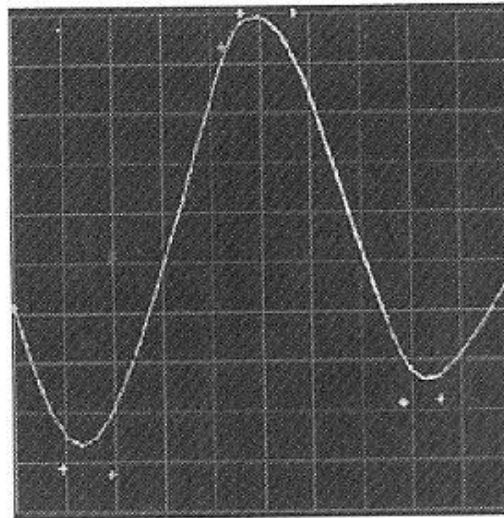
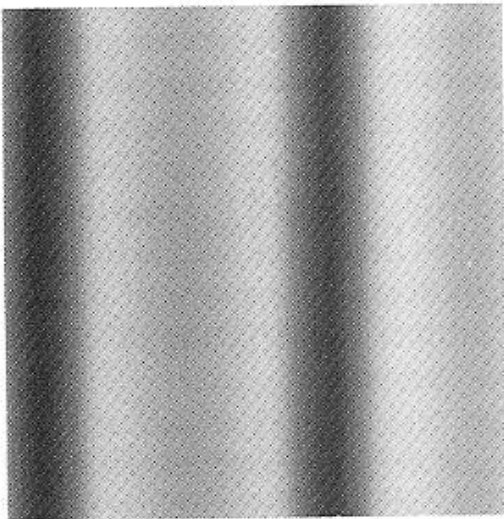


Synthesis of Turbulence (2D)



Example: Marble Texture Function

- **Overall structure: alternating layers of white and colored marble**
 - $f_{\text{marble}}(x,y,z) := \text{marble_color}(\sin(x))$
 - `marble_color` : transfer function (see lower left)
- **Realistic appearance: simulated turbulence**
 - $f_{\text{marble}}(x,y,z) := \text{marble_color}(\sin(x+\text{turbulence}(x,y,z)))$
- **Moving object: turbulence function also transformed**



Further Procedural Texturing Applications

- **Bark**
 - Turbulated sawtooth function
 - Bump mapping
- **Clouds**
 - White blobs
 - Turbulated transparency along edge
 - Transparency mapping
- **Animation**
 - Vary procedural texture function's parameters over time

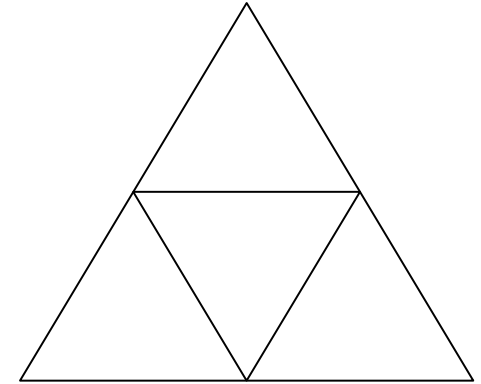
Fractal Landscapes

- **Procedural generation of geometry**
- **Complex geometry at virtually no memory cost**
 - Can be difficult to ray trace !!



Fractal Landscapes

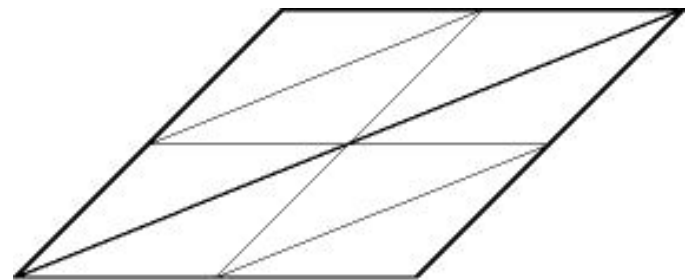
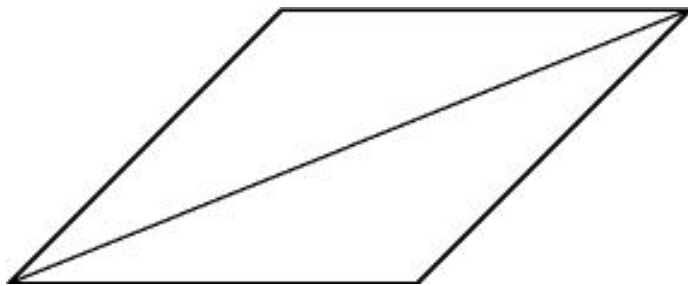
- **Coarse triangle mesh approximation**
- **1:4 triangle subdivision**
 - Vertex insertion at edge-midpoints
- **New vertex perturbation**
 - Random displacement along normal
 - Scale of perturbation depends on subdivision level
 - Decreasing power spectrum
 - Parameter models surface roughness
- **Recursive subdivision**
 - Level of detail (LOD) determined by # subdivisions
- **All done inside renderer !**
 - LOD generated locally when/where needed (bounding box test)
 - Minimal I/O cost (coarse mesh only)



Fractal Landscapes

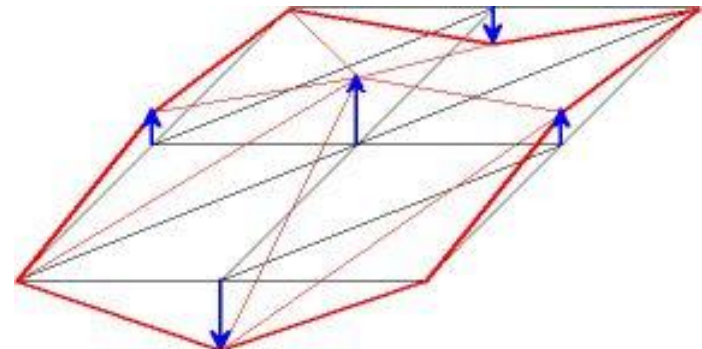
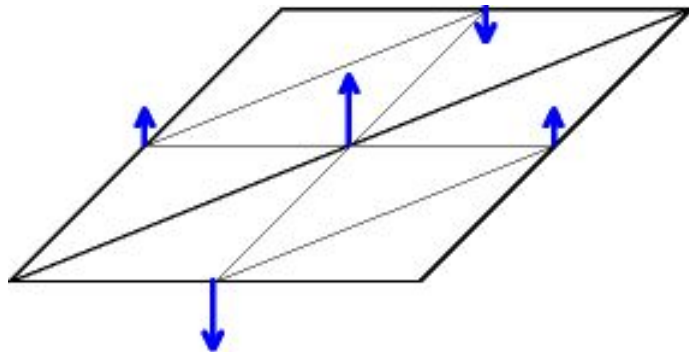
- **Triangle subdivision**

- Insert new vertices at edge midpoints
- 1:4 triangle subdivision



- **Vertex displacement**

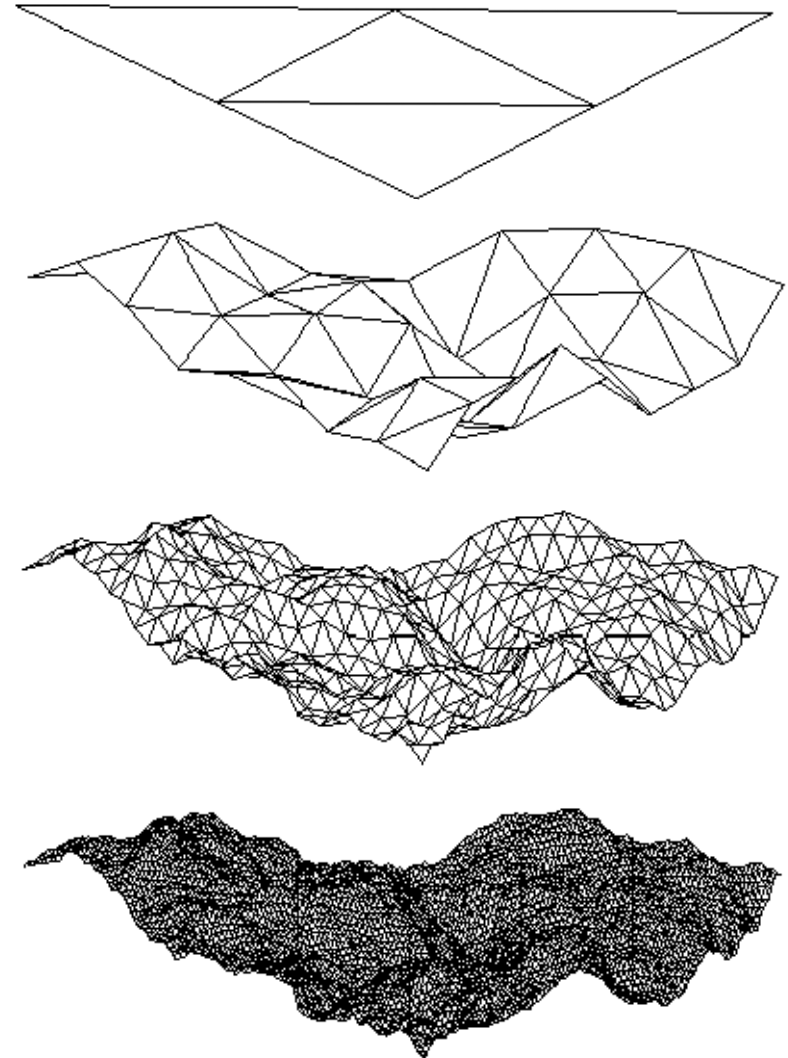
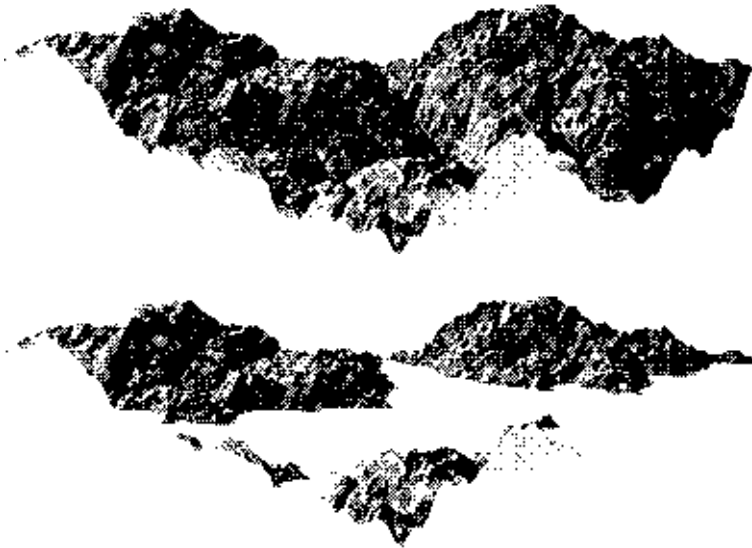
- Along original triangle normal



Courtesy <http://www.uni-paderborn.de/SFB376/projects/a2/zBufferMerging/>

Fractal Landscape Generation

- **Base mesh**
- **Repeated subdivision & vertex displacement**
- **Shading + Water surface + Fog + ...**



Fractal Landscape Ray Tracing

- **Fractal terrain generated on-the-fly**
- **Problem: where is the ray-surface interaction ?**
 - Triangle mesh not a-priori known
- **Solution: bounding boxes**
 - Maximum possible bounding box around each triangle
 - Decreasing displacement amplitude: finite bounding box
- **Algorithm**
 - Intersect ray with bounding box
 - If hit, subdivide corresponding triangle
 - Compute bounding boxes of 4 new triangles
 - Test against 4 new bounding boxes
 - Iterate until termination criterion fulfilled (LOD / pixel size)