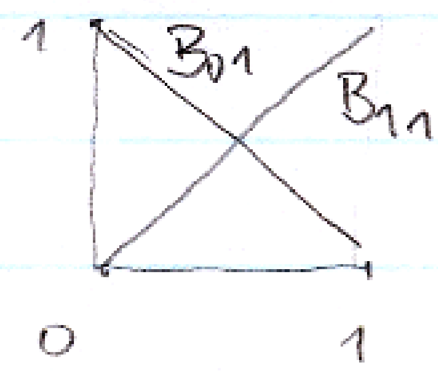
Bernsteinove polynómy - grafy $n=1, n=2, n=3$

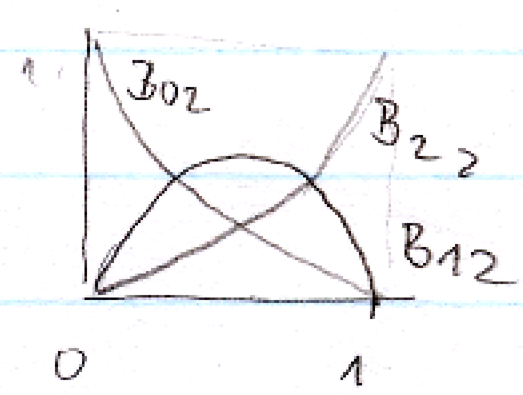
$n=1$ $\qquad B_{i1}(t) = \binom{1}{i} t^i (1-t)^{1-i}$

$$B_{01}(t) = 1-t$$
$$B_{11}(t) = t$$



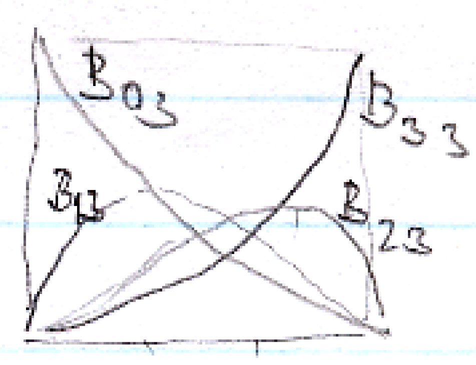
$n=2$ $\qquad B_{i2}(t) = \binom{2}{i} t^i (1-t)^{2-i}$

$$B_{02}(t) = (1-t)^2$$
$$B_{12}(t) = 2t(1-t)$$
$$B_{22}(t) = t^2$$



$n=3$ $\qquad B_{i3}(t) = \binom{3}{i} t^i (1-t)^{3-i}$

$$B_{03}(t) = (1-t)^3$$
$$B_{13}(t) = 3t(1-t)^2$$
$$B_{23}(t) = 3t^2(1-t)$$
$$B_{33}(t) = t^3$$



Vlastnosti - Vety

• 1 $\boxed{\text{nezápornosť}}$: $B_{in}(t) \geq 0$ pre každé $i, n, t \in \langle 0,1 \rangle$

• 2 $\boxed{\text{rozklad jednotky}}$: $\sum_{i=0}^{n} B_{in}(t) = 1$ pre každé $t \in \langle 0,1 \rangle$

Dôkaz: $\sum_{i=0}^{n} B_{in}(t) = \sum_{i=0}^{n} \binom{n}{i} t^i (1-t)^{n-i} = ((1-t)+t)^n = 1$

• 3 $\boxed{B_{0n}(0) = B_{nn}(1) = 1}$; inak $B_{in}(0) = 0$ pre $i = 1, \dots, n$
a $B_{in}(1) = 0$ pre $i = 0, \dots, n-1$

• 4 $\boxed{\text{symetria}}$: $B_{in}(1-t) = B_{n-i,n}(t) \quad i = 0, \dots, n$

Dôkaz: $B_{in}(1-t) = \binom{n}{i}(1-t)^i (1-(1-t))^{n-i} = \binom{n}{i}(1-t)^i t^{n-i}$

$$\parallel$$

$B_{n-i,n}(t) = \binom{n}{n-i} t^{n-i} (1-t)^{n-(n-i)} = \binom{n}{n-i} t^{n-i} (1-t)^i$

$\binom{n}{i} = \binom{n}{n-i}$

• 5. **rekurzia** (rekurentný vzorec)

$$B_{in}(t) = (1-t)\, B_{i,n-1}(t) + t\, B_{i-1,n-1}(t)$$

$$i = 0, \dots, n; \quad B_{-1,n-1}(t) = 0$$

$$B_{n,n-1}(t) = 0$$

Dôkaz:

$$B_{in}(t) = \binom{n}{i} t^i (1-t)^{n-i} = \left[\binom{n-1}{i} + \binom{n-1}{i-1}\right] t^i (1-t)^{n-i} =$$

$$= \binom{n-1}{i} t^i (1-t)^{n-i} + \binom{n-1}{i-1} t^i (1-t)^{n-i} =$$

$$= (1-t)\left[\binom{n-1}{i} t^i (1-t)^{n-i-1}\right] + t\left[\binom{n-1}{i-1} t^{i-1} (1-t)^{n-i}\right]$$

$$= (1-t)\, B_{i,n-1}(t) + t\, B_{i-1,n-1}(t)$$

• 6. **derivácie**

1. derivácia: $B'_{in}(t) = n\{B_{i-1,n-1}(t) - B_{i,n-1}(t)\}$

Dôkaz: $B'_{in}(t) = \left[\binom{n}{i} t^i (1-t)^{n-i}\right]' = i\binom{n}{i} t^{i-1}(1-t)^{n-i} - (n-i)\binom{n}{i} t^i (1-t)^{n-i-1}$

$$= \frac{i \cdot n!}{i!\,(n-i)!} t^{i-1}(1-t)^{n-i} - \frac{(n-i)\, n!}{i!\,(n-i)!} t^i (1-t)^{n-i-1} =$$

$$= \frac{n\,(n-1)!}{(i-1)!\,(n-i)!} t^{i-1}(1-t)^{n-i} - \frac{n\,(n-1)!}{i!\,(n-i-1)!} t^i (1-t)^{n-i-1} =$$

$$= n\, B_{i-1,n-1}(t) - n\, B_{i,n-1}(t) = n\{B_{i-1,n-1}(t) - B_{i,n-1}(t)\}$$

$$B_{-1,n-1}(t) = B_{n,n-1}(t) = 0$$

2. derivácia

$$B''_{in}(t) = n\,(n-1)\{B_{i-2,n-2}(t) - 2B_{i-1,n-2}(t) + B_{i,n-2}(t)\}$$

• 7. $\boxed{B_{in}(t)}$ má práve jeden lokálny extrém – **lokálne maximum** pre $\boxed{t = \frac{i}{n}}$

Dôkaz

$$B'_{in}(t) = i\binom{n}{i} t^{i-1}(1-t)^{n-i} - (n-i)\binom{n}{i} t^i (1-t)^{n-i-1} = 0$$

$$t^{i-1} (1-t)^{n-i-1}\left[i\binom{n}{i}(1-t) - (n-i)\binom{n}{i} t\right] = 0$$

$$i\binom{n}{i}(1-t) - (n-i)\binom{n}{i} t = 0$$

$$i(1-t) - (n-i)t = 0 \Rightarrow t = \frac{i}{n}$$