Image preprocessing

Mathematical morphology - basics

Morphological operations

- Non-linear algebra operations
- simplify images, and quantify and preserve the main shape characteristics of objects.

Used for

- Image preprocessing.
- Enhancing object structure.
- Segmenting objects from the background.
- Quantitative description of objects.

- Binary images
- Image as point sets -
- Point is a pair of integers
- $image \subset \mathbb{Z}^2$

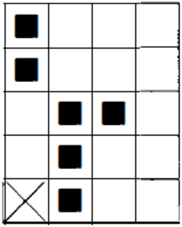
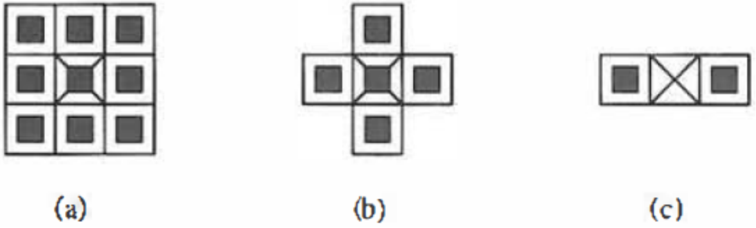


Figure 13.1: A point set example.

- Standard set operations $\cap$, $\cup$, $\subset$, ^{c}
- Set difference $X \backslash Y = X \cap Y^c$.
- Points belonging to objects represents set X.
- Points of the complement X^c represents background.

Morphological transformation

- is given by relation of the image (point set X) with another small point set B – structuring element.



- B is expressed with respect to local origin (representative point) $\mathcal{O}$

- Dual morphological operation Ψ^* to morphological operation Ψ

$$\Psi(X) = (\Psi^*(X^c))^c$$

- Translation X_h of point set X by vector h

$$X_h = \{p \in \mathcal{E}^2, p = x + h \text{ for some } x \in X\}$$

Quantitative morphological operation

- Compatible with translation

$$\Psi_{\mathcal{O}}(X_h) = (\Psi_{-h}(X))_h \qquad \text{or} \qquad \Psi(X_h) = \Psi(X)_h$$

- Compatible with change of scale

$$\Psi_\lambda(X) = \lambda\Psi(\frac{1}{\lambda}X) \qquad \text{or} \qquad \Psi(\lambda X) = \lambda\Psi(X)$$

Quantitative morphological operation

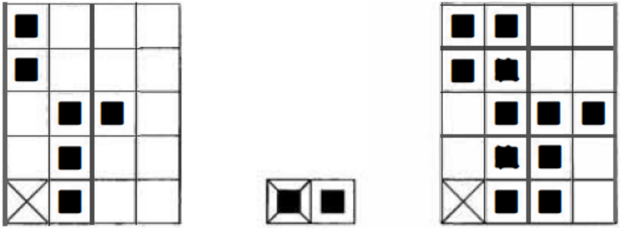
- Local knowledge : for any bounded $Z' \subset \Psi(X)$ there exists a bounded set Z such that

$$(\Psi(X \cap Z)) \cap Z' = \Psi(X) \cap Z'$$

- Upper semi-continuity

Binary dilation

$$X \oplus B = \{p \in \mathrm{E}^2; p = x + b, x \in X \text{ and } b \in B\}$$

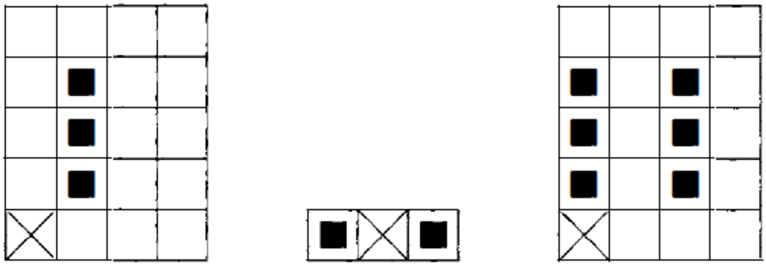


Binary dilation – properties

- $X \oplus Y = Y \oplus X$
- $X \oplus (B \oplus D) = (X \oplus B) \oplus D$
- $X \oplus B = \bigcup_{b \in B} X_b$
- $X_h \oplus B = (X \oplus B)_h$
- If $X \subset Y$ then $X \oplus B \subset Y \oplus B$

Binary dilation

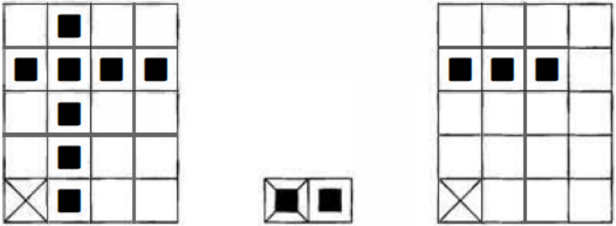
Dilation where the representative point is not a member of the structuring element



Binary dilation – Example

Erosion

$$X \ominus B = \{p \in \mathcal{E}^2; p = x + b \in X \text{ for all } \ b \in B\}$$



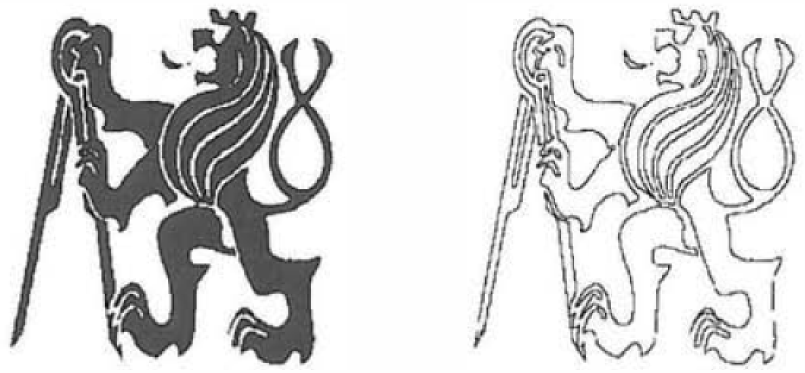
- Dual operator of dilation

Erosion

- $X \ominus B = \{p \in \mathcal{E}^2; B_p \subseteq X\}$
- $X \ominus B = \bigcap_{b \in B} X_{-b}$
- $X_h \ominus B = (X \ominus B)_h$
- $X \ominus B_h = (X \ominus B)_{-h}$
- If $X \subset Y$ then $X \ominus B \subset Y \ominus B$
- If $D \subset B$ then $X \ominus B \subset X \ominus D$

Binary erosion – Example

Binary erosion – Example



Opening and closing

Opening – Erosion followed by dilation

$$X \circ B = (X \ominus B) \oplus B$$

Closing – Dilation followed by erosion

$$X \bullet B = (X \oplus B) \ominus B$$

Closing

- Connects objects that are close to each other.
- Fills up small holes.
- Smoothes the object outline.

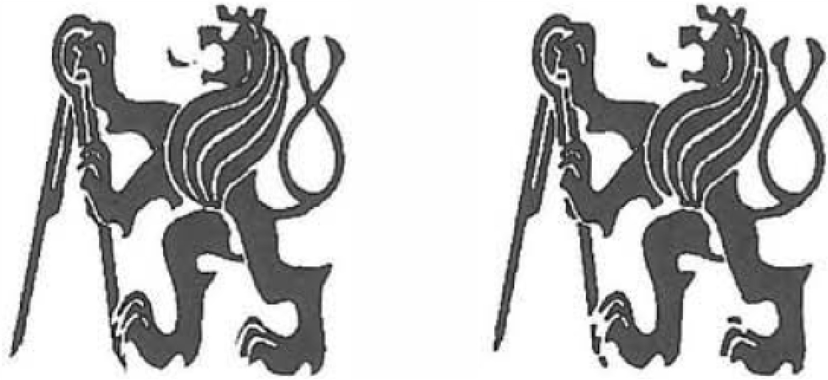
Opening and closing are dual operations

$$(X \bullet B)^c = X^c \circ \breve{B}$$

and idempotent

$$X \circ B = (X \circ B) \circ B$$
$$X \bullet B = (X \bullet B) \bullet B$$

Opening – Example



Closing – Example

Grey-scale dilation and erosion

Top surface

- $A \subseteq \mathcal{E}^n$
- $F = \{x \in \mathcal{E}^{n-1} \text{ for some } y \in \mathcal{E}, (x, y) \in A\}$ — support of A
- Top surface of A, $T[A] : F \to \mathcal{E}$

$$T[A](x) = \max\{y, (x, y) \in A\}$$

Grey-scale dilation and erosion

Umbra

- $F \subseteq \mathcal{E}^{n-1}$
- $f : F \to \mathcal{E}$
- Umbra of f, $U[f] \subseteq F \times \mathcal{E}$

$$U[f] = \{(x, y) \in F \times \mathcal{E}; y \leq f(x)\}$$

Grey-scale dilation and erosion

- $F, K \subseteq \mathcal{E}^{n-1}$
- $f : F \to \mathcal{E}$, $k : K \to \mathcal{E}$
- Dilation $\oplus$ of f by k, $f \oplus k : F \oplus K \to \mathcal{E}$

$$f \oplus k = T[U[f] \oplus U[k]]$$

Computationally

$$(f \oplus k)(x) = \max\{f(x-z) + k(z), z \in k, x - z \in F\}$$

Grey-scale dilation and erosion

- $F,\ K \subseteq \mathcal{E}^{n-1}$
- $f : F \to \mathcal{E},\ k : K \to \mathcal{E}$
- Erosion $\ominus$ of f by k, $f \ominus k : F \ominus K \to \mathcal{E}$

$$f \ominus k = T[U[f] \ominus U[k]]$$

Computationally

$$(f \oplus k)(x) = \min_{z \in K}\{f(x-z) - k(z)\}$$

The umbra homeomorphism theorem

- $F,\ K \subseteq \mathcal{E}^{n-1}$
- $f : F \to \mathcal{E},\ k : K \to \mathcal{E}$

$$U[f \oplus k] = U[f] \oplus U[k]$$
$$U[f \ominus k] = U[f] \ominus U[k]$$

Consequence: derivation of properties of grey-scale morphology from binary morphology

Grey-scale opening and closing

Gray-scale opening

$$f \circ k = (f \ominus k) \oplus k$$

Gray-scale closing

$$f \bullet k = (f \oplus k) \ominus k$$

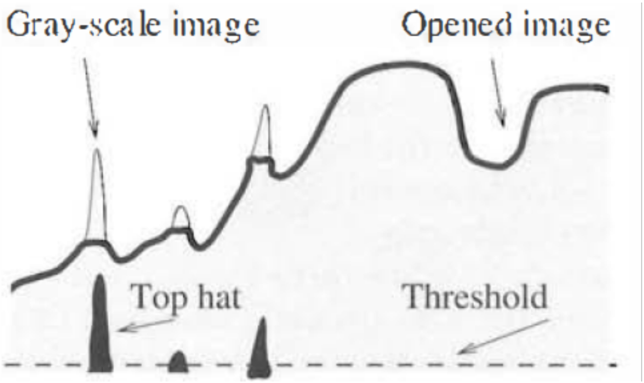
Top-hat transform

- X – grey-level image
- K – structuring element
- Top-hat transform

$$X \setminus (X \circ K)$$

Good for extracting light (dark) object from dark (light) slowly changing background.

Top-hat transform



Hit or miss

- Composite structuring element $\mathcal{B} = (B_1, B_2)$ – pair of disjoint sets
- Hit-or-miss transformation $\otimes$

$$X \otimes \mathcal{B} = \{x; B_1 \subseteq X \text{ and } B_2 \subseteq X^c\}$$

- $X \otimes B = (X \ominus B_1) \cap (X^c \ominus B_2) = (X \ominus B_1) \backslash (X \oplus \breve{B}_2)$

Thinning and thickening

- X – image
- $B = (B_1, B_2)$ – composite structuring element
- Thinning
$$X \oslash B = X \backslash (X \otimes B)$$
- Thickening
$$X \odot B = X \cup (X \otimes B)$$